

MONAN Dynamics

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WMO Workshop - Nov 2025

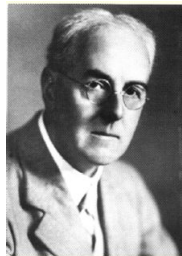


Summary

- ❏ Historical background
- ❏ Basic concepts of MONAN/MPAS dynamics
- ❏ Unstructured grids theory
- ❏ MONAN computational workflow

Summary

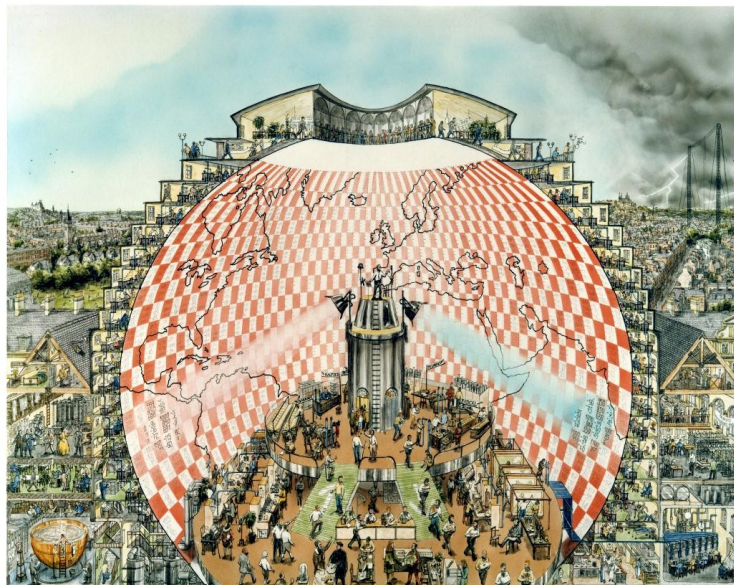
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- MONAN computational workflow



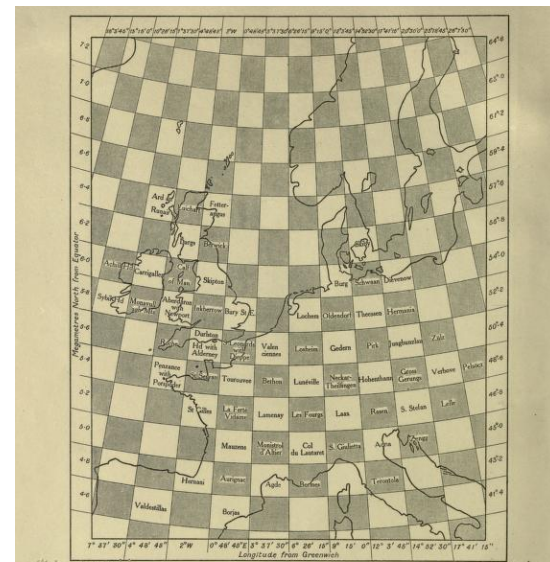
History of Weather Forecasting

Lewis Fry Richardson (UK 1881 -1953)

- Richardson, L.F., 1922. Weather prediction by numerical process. Cambridge university press.
- Primitive equations, spherical coordinates, finite differences, resolution ~ 200km (hand calculation during WW-I)



"Weather Forecasting Factory" by Stephen Conlin, 1986.

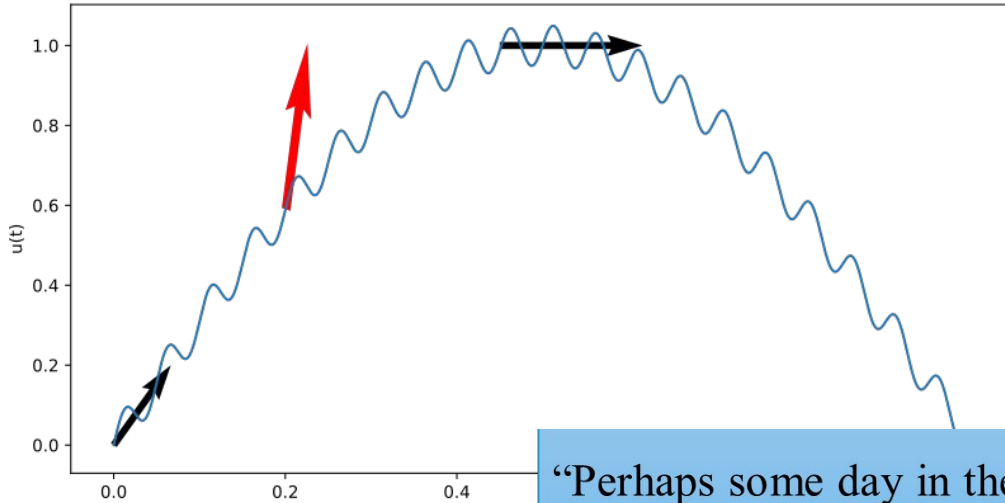


Hunt, J.C., 1998. Lewis Fry Richardson and his contributions to mathematics, meteorology, and models of conflict. Annual Review of Fluid Mechanics.

Richardson's Results

- Predicted a 145mb change over 6 hours at a grid point
- Observations showed almost no pressure change

Slow-Fast Interaction

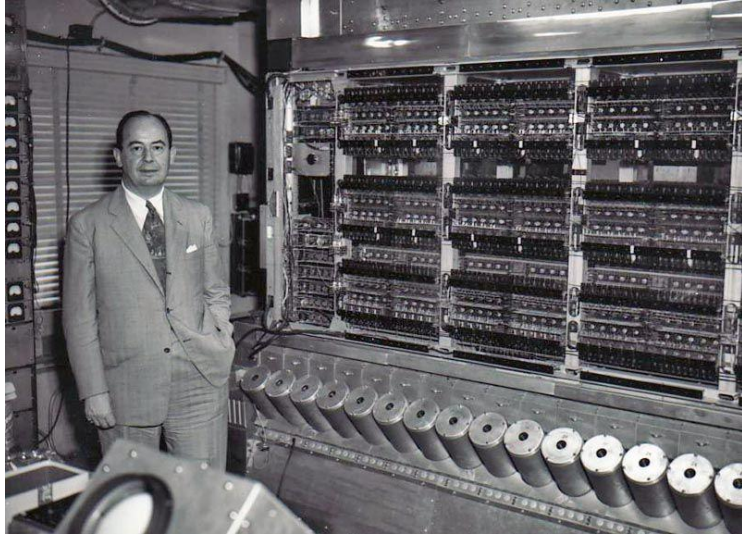


- **Great ideas, but the dynamics is multiscale!**
- **Model initialization issues (initial imbalance between pressure and wind)**

“Perhaps some day in the dim future it will be possible to advance the computations faster than the weather advances”

Lynch, P., 1999. Richardson's marvelous forecast. In The life cycles of extratropical cyclones. American Meteorological Society, Boston, MA.

First successful numerical weather prediction



John von Neumann (1903 - 1957)

Meteorological Program, Princeton (1946):

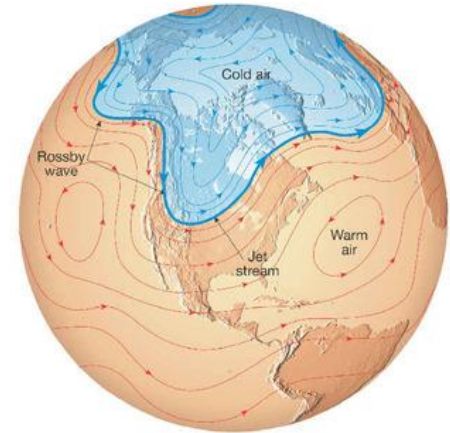
- Jule Gregory Charney, Philip Thompson, Larry Gates, Ragnar Fjørtoft, Klara Dan von Neumann.
- ENIAC (Electronic Numerical Integrator and Computer) - 20,000 vacuum tubes - 100 kHz clock

Charney, J.G., Fjørtoft, R. and Neumann, J., 1950. Numerical Integration of the Barotropic Vorticity Equation. *Tellus Series A*, 2, pp.237-254.

$$\frac{D\eta}{Dt} = 0$$

$$\eta = \zeta + f$$

Conservation of absolute vorticity
(Relative + Coriolis) along with flow



Thompson, P.D., 1983. A history of numerical weather prediction in the United States. *Bulletin of the American Meteorological Society*, 64(7)

Early forecasts

- 1954: Rossby and team produced the first operational forecast in Sweden based on the barotropic equation.
- 1955-56: Charney, Thompson, Gates and team: Operational numerical weather prediction in the United States with layered barotropic models.
- 1959: Operational weather forecast in Japan

60's: **Primitive equations** are back
(with improved initialization of the models)

Climate change modelling started!

- Late 60s: Manabe, Wetherald, Smagorinsky et al. at GFDL-NOAA
- 1970s: First coupled models (atmosphere, ocean)

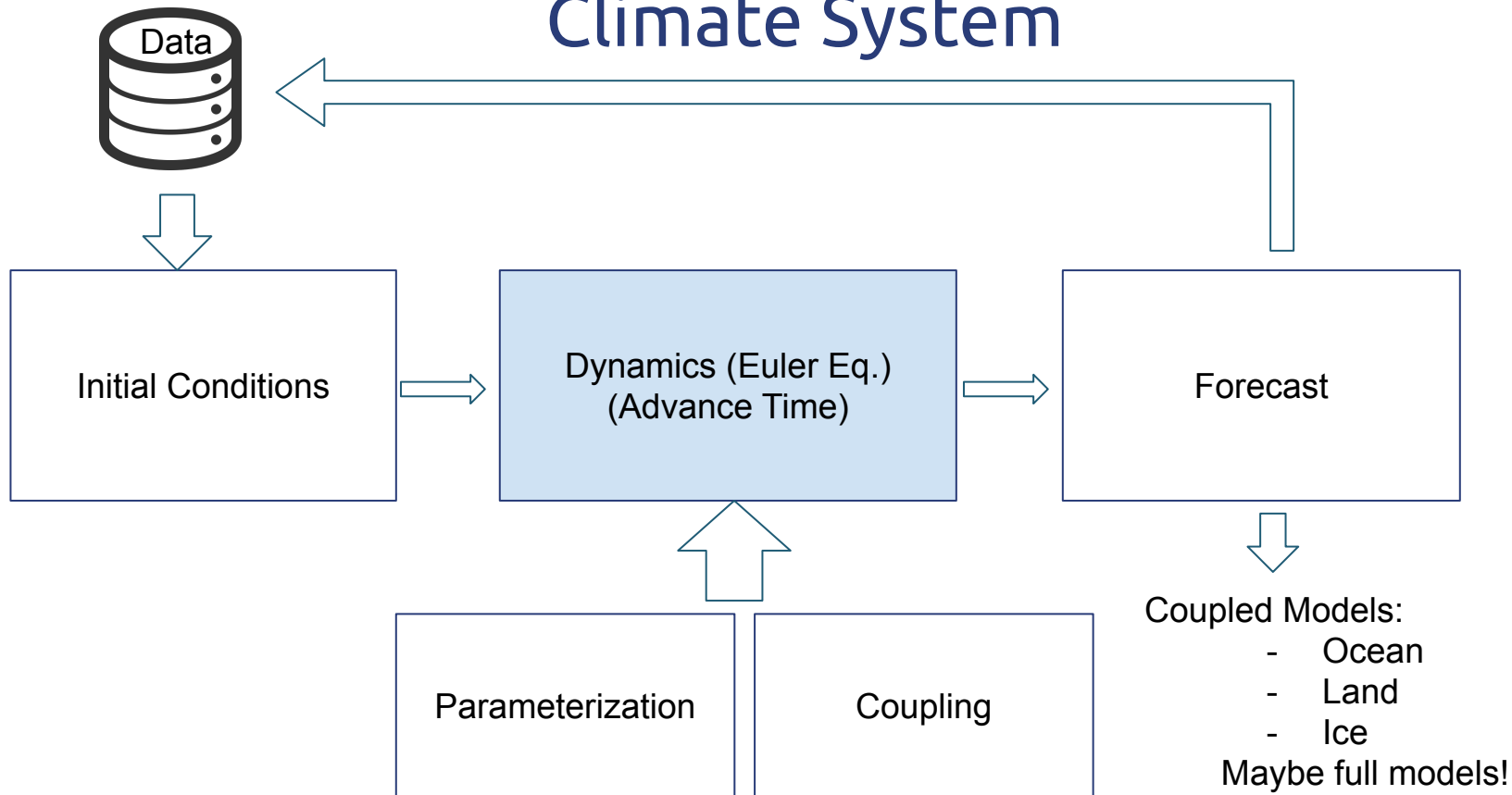


Numerically solve the equations on the whole sphere

Randall, D.A., Bitz, C.M., Danabasoglu, G., Denning, A.S., Gent, P.R., Gettelman, A., Griffies, S.M., Lynch, P., Morrison, H., Pincus, R. and Thuburn, J., 2019. 100 Years of Earth System Model Development. *Meteorological Monographs*, 59.

Science Museum: <https://www.sciencemuseum.org.uk/objects-and-stories/weather-forecasting-and-climate-modelling-short-history>

Operational Weather Forecast / Climate System



The computational challenge

Underlying dynamics equations are simplifications of

EGEON/INPE (~4k cores)

$$\frac{D\mathbf{u}}{Dt} = -2\boldsymbol{\Omega} \times \mathbf{u} - \frac{1}{\rho} \nabla p + \mathbf{g} + \mathbf{F}_r \text{ (Momentum)}$$

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u} \text{ (Continuity)}$$

$$c_v \frac{DT}{Dt} = -\frac{\rho}{\rho} \nabla \cdot \mathbf{u} \text{ (Thermodynamics)}$$

+ All parametrizations

- $\mathbf{u} = (u, v, w)$: wind velocity
- p : pressure
- ρ : density
- T : temperature
- $D/Dt = \partial/\partial t + \mathbf{u} \cdot \nabla$

...to be solved on the whole sphere...



<https://projetos.cptec.inpe.br/attachments/download/15461/Guia%20o%20Usu%C3%A1rio%20EGEON%20INPE.pdf>

... on a supercomputer.

Spectral Models

Emerged around 1960-1970.

Main concept: Derivatives are calculated in spectral space

Requires: Spherical harmonics with Fast Fourier Transform and “Fast” Legendre transforms global models

Pseudo-spectral: Global coupling of points.

Operational Examples:

- IFS-ECMWF (European):
 <10km
- BAM-CPTEC-INPE (Brazil):
 ~20km
- GFS-NOAA (USA) - up to 2019:
 ~13km

◆ **Resolution increases of the deterministic 10-day medium-range Integrated Forecast System (IFS) over ~28 years at ECMWF:**

- ◆ 1983: T 63 (~316km)
- ◆ 1987: T 106 (~188km)
- ◆ 1991: T 213 (~95km)
- ◆ 1998: T_L319 (~63km)
- ◆ 2000: T_L511 (~39km)
- ◆ 2006: T_L799 (~25km)
- ◆ 2010: T_L1279 (~16km)
- ◆ 2015: T_L2047 (~10km) **Hydrostatic, parametrized convection**
- ◆ 2020-???: (~1-10km) **Non-hydrostatic, explicit deep convection, different cloud-microphysics and turbulence parametrization, substantially different dynamics-physics interaction...**



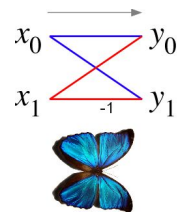
IFS Cycle 49r1 - Nov 2024

	Component	Horizontal resolution		Vertical resolution [levels]
Atmosphere	HRES	01280	~9 km	137
	ENS	01280	~9 km	137
	ENS extended	0320	~36 km	137

<https://confluence.ecmwf.int/display/FCST/Implementation+of+IFS+Cycle+49r1>

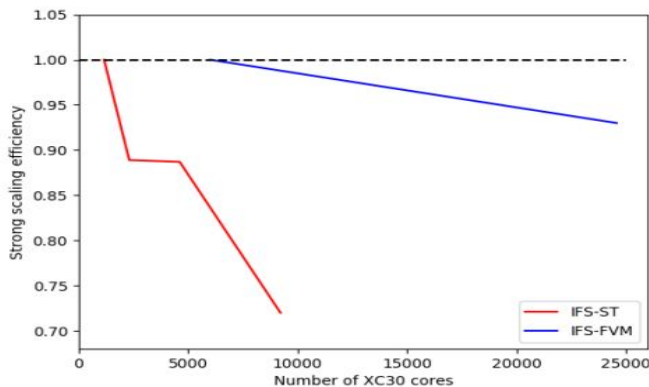
Why change? —> Scalability

Drawbacks:



- Legendre Transform is $O(N^2)$ operations
-> use “fast” Legendre transforms (Butterfly Algorithm)
- Spectral transforms require global communication
-> reduces scalability (parallelism has communication bottlenecks)

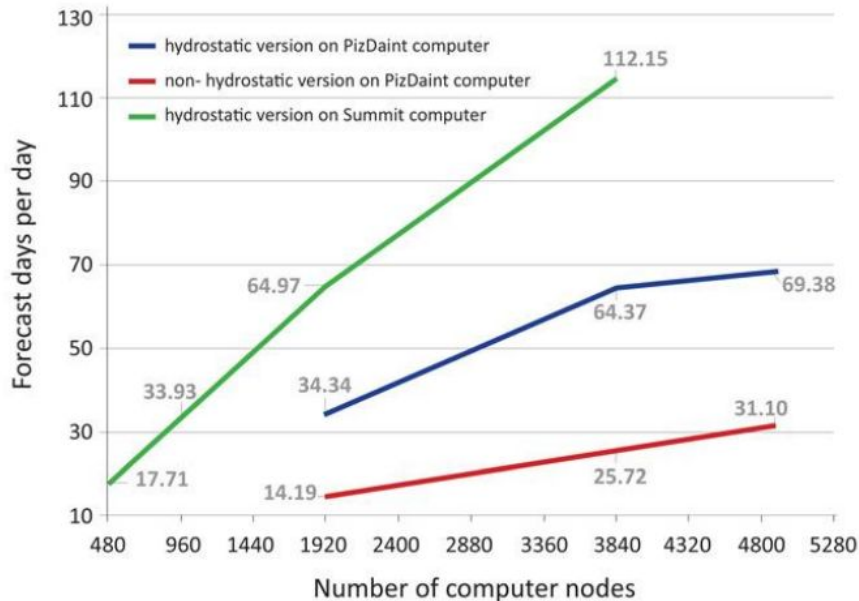
IFS-ECMWF Example:



Strong scaling

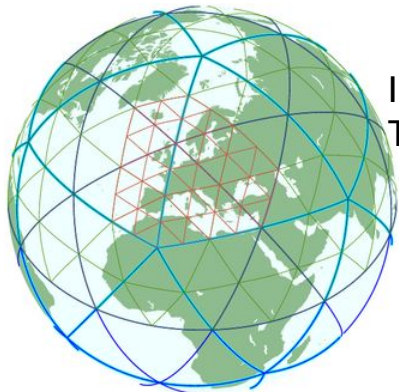
Time-to-solution

Speed comparison using: high-resolution ECMWF IFS
1.25 km, 62-vertical levels (TCO7999)

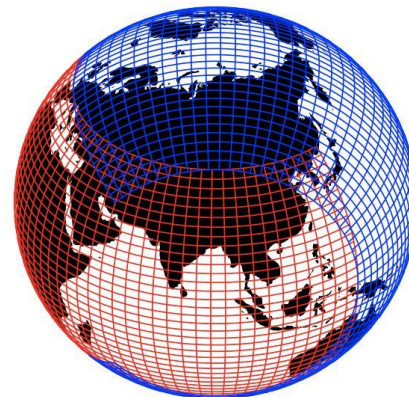


Isotropic Alternatives ~ 2000-2010

Cubed Sphere (CAM-SE/FV3/NUMA-USA)

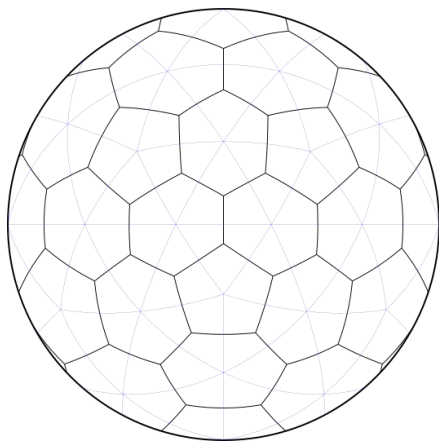


Icosahedral/
Triangular
(ICON-Germany)

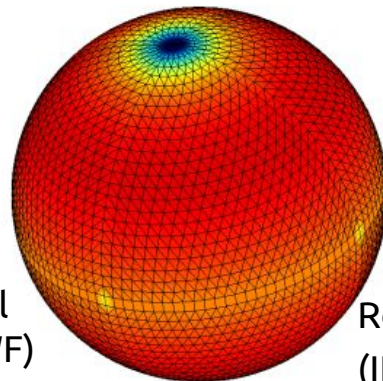


Yin-Yang
(GEM-Canada)

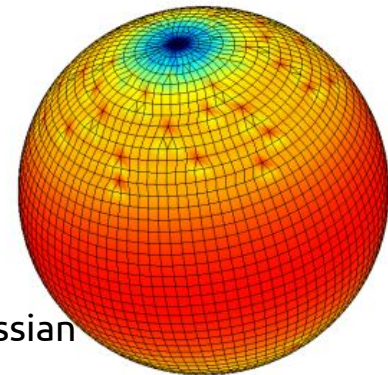
Voronoi
(MPAS/FIM/
OLAM-USA
NICAM-Japan)



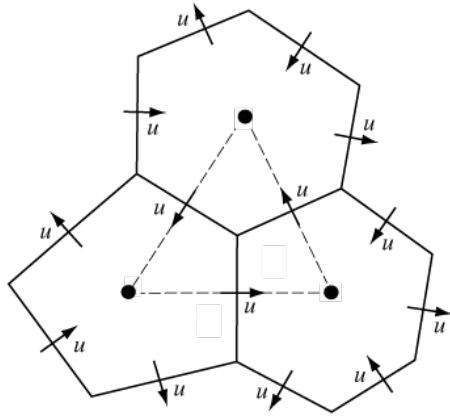
Octahedral
(IFS-ECMWF)



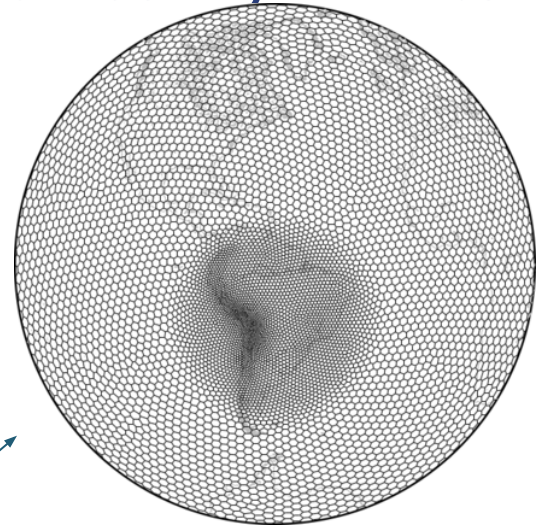
Reduced Gaussian
(IFS-ECMWF)



Model for Prediction Across Scales Dynamics

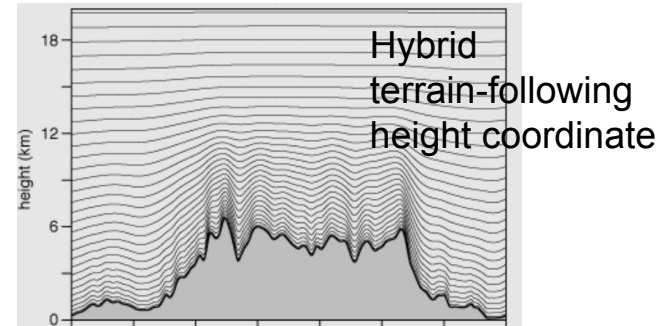


NCAR and Los Alamos Nat Lab



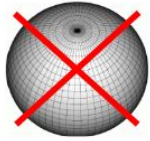
Compressible nonhydrostatic equations

C-grid staggered variables on the horizontal Voronoi mesh. Normal velocities are defined on the cell faces and all other scalar variables are defined at the cell centers. Vertical vorticity is defined at the cell vertices. (from: <https://mpas-dev.github.io/>)



- Skamarock, W.C., Klemp, J.B., Duda, M.G., Fowler, L.D., Park, S.H. and Ringler, T.D., 2012. A multiscale nonhydrostatic atmospheric model using centroidal Voronoi tessellations and C-grid staggering. Monthly Weather Review, 140(9)

MPAS Development



2005 Global lat-lon (WRF) problematic.



2006 Triangles - problems with divergence.

Yin-Yang: local conservation past 1st-order accuracy?

2007 Cubed-sphere: Corner point problems?



Hex grid: C-grid problem solved for perfect hex mesh.

2008 C-grid problem solved for general Voronoi mesh.

2009 Unstructured-mesh MPAS SW eqns. solver.

MPAS hydrostatic eqns. solver.

2010 MPAS nonhydrostatic eqns. solver.

Hydrostatic MPAS in CAM/CESM.



2011 WRF-NRCM physics in MPAS.

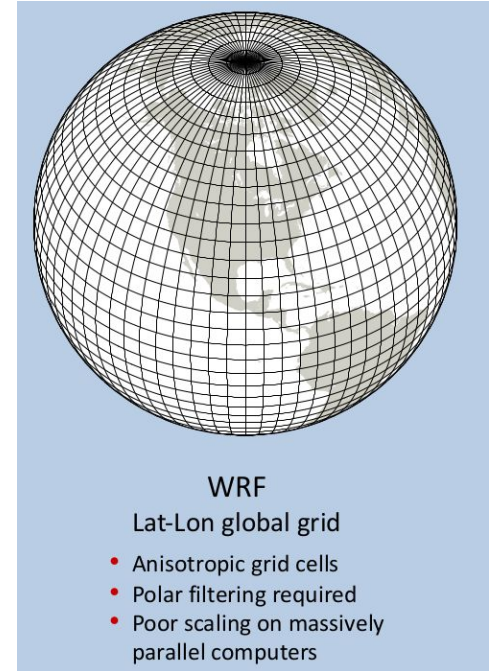
2012 DART data assimilation.

3km global mesh tests on Yellowstone.

2013 MPAS V1.0 release (atmosphere, ocean)

MPAS-Atmosphere real-time TC forecast testing.

2014 Scale-aware physics testing begins.



WRF

Lat-Lon global grid

- Anisotropic grid cells
- Polar filtering required
- Poor scaling on massively parallel computers

MONAN

MONAN – Model for Ocean-laNd-Atmosphere predictioN

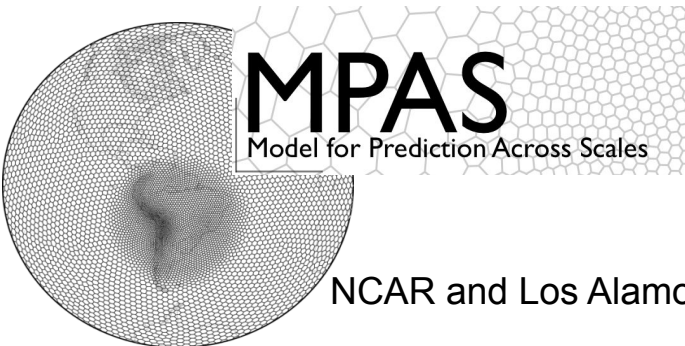
Modelo para Previsão dos Oceanos, Superfícies Terrestres e Atmosfera



Model for Prediction Across Scales Dynamics

9th meeting of the Scientific Committee of MONAN (03/08/2023):

- MPAS as basis for MONAN
- Comparison between MPAS FV3/Shield and GEF
- Committee decided to have MPAS-Atmosphere as initial basis for MONAN-Atmosphere.



NCAR and Los Alamos Nat Lab

Summary

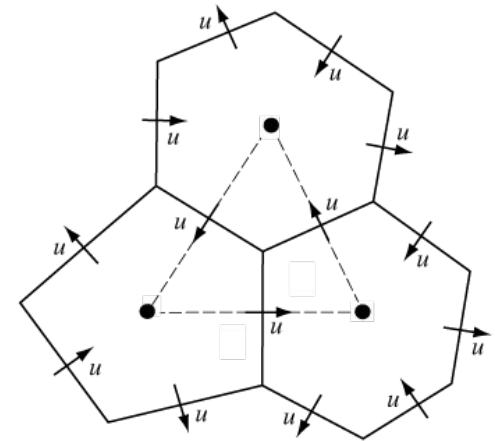
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Dynamics: Key references

- **Horizontal dynamics:** Ringler, T., J. Thuburn, J. Klemp and W. Skamarock, 2010: A unified approach to energy conservation and potential vorticity dynamics on arbitrarily structured C-grids, Journal of Computational Physics.
- **Time integration:** Wicker, L.J. and Skamarock, W.C., 2002. Time-splitting methods for elastic models using forward time schemes. Monthly weather review, 130(8), pp.2088-2097.
- **Advection:** Skamarock, W.C. and Gassmann, A., 2011. Conservative transport schemes for spherical geodesic grids: High-order flux operators for ODE-based time integration. Monthly Weather Review, 139(9), pp.2962-2975.
- **Vertical:** Klemp, J.B., Skamarock, W.C. and Dudhia, J., 2007. Conservative split-explicit time integration methods for the compressible nonhydrostatic equations. Monthly Weather Review, 135(8), pp.2897-2913.
- **3D Model:** Skamarock, W.C., Klemp, J.B., Duda, M.G., Fowler, L.D., Park, S.H. and Ringler, T.D., 2012. A multiscale nonhydrostatic atmospheric model using centroidal Voronoi tessellations and C-grid staggering. Monthly Weather Review, 140(9), pp.3090-3105.

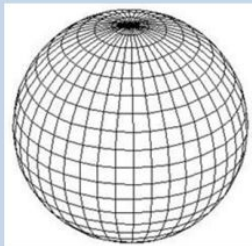
Following content comes mostly from MPAS tutorials given by NCAR!

Full tutorials here: <https://github.com/CGFD-USP/MPAS-References>

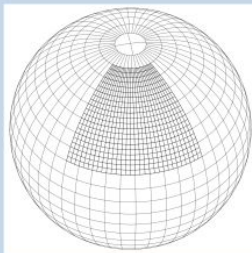


WRF vs MPAS

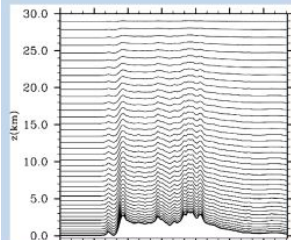
WRF Characteristics



- Lat-Lon global grid
 - Anisotropic grid cells
 - Polar filtering required
 - Poor scaling on massively parallel computers

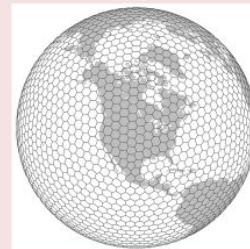


- Grid refinement through domain nesting
 - Flow distortions at nest boundaries

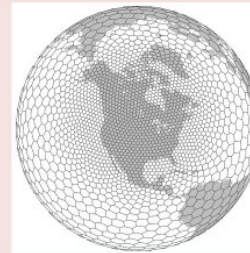


- Pressure-based terrain-following sigma vertical coordinate

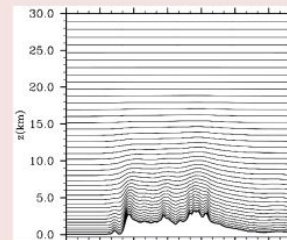
MPAS Characteristics



- Unstructured Voronoi (hexagonal) grid
 - Good scaling on massively parallel computers
 - No pole problems

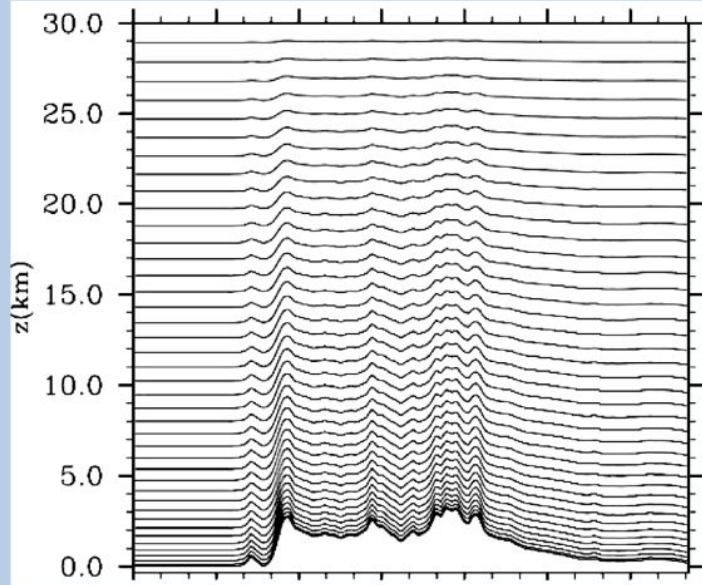


- Smooth grid refinement on a conformal mesh
 - Increased accuracy and flexibility in varying resolution

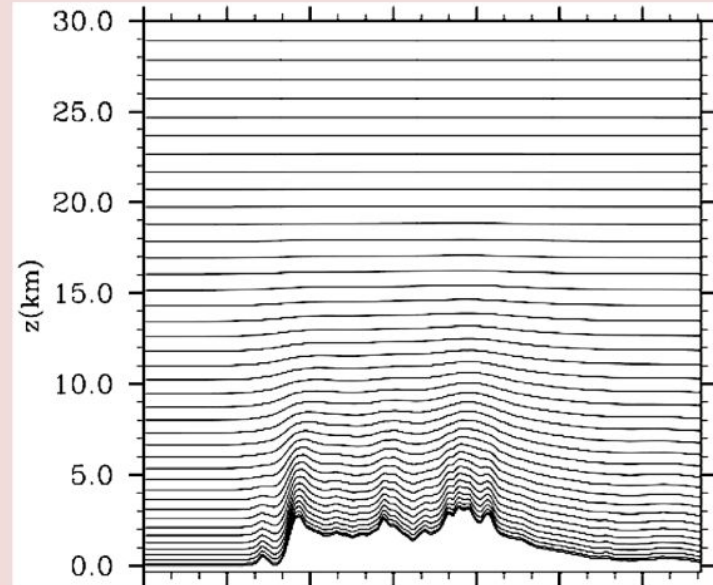


- Height-based hybrid smoothed terrain-following vertical coordinate
 - Improved numerical accuracy

Vertical Coordinates



WRF
Pressure-based
terrain-following sigma
vertical coordinate



MPAS
Height-based hybrid smoothed
terrain-following vertical
coordinate

- Improved numerical accuracy

Vertical Coordinate

- Height hybrid terrain-following

$A = 0$ Pure height coordinate.

$A = 1 - \zeta/z_t$ Pure terrain following

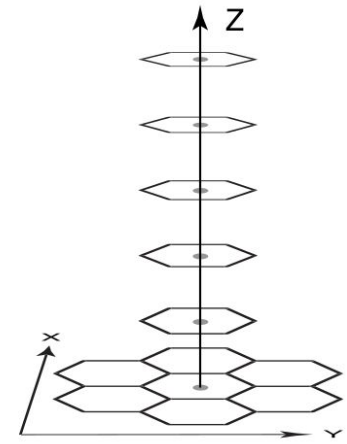
$$z = \zeta + A(\zeta)h_s(x, y, \zeta)$$



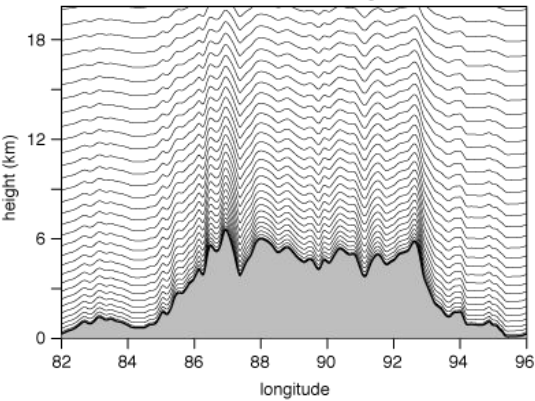
Smoothed terrain height

Factor Height/Terrain Following

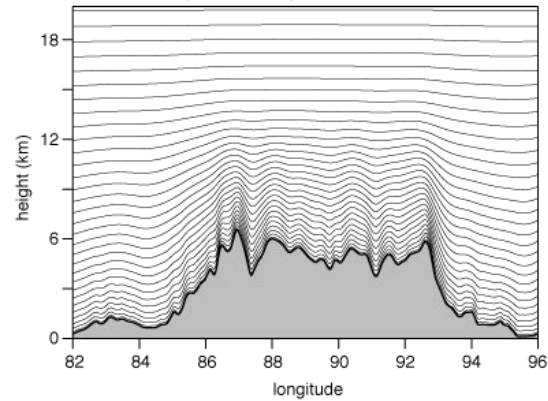
Nominal heights



Traditional Terrain-Following Coordinate



Klemp (2011) Hybrid Coordinate

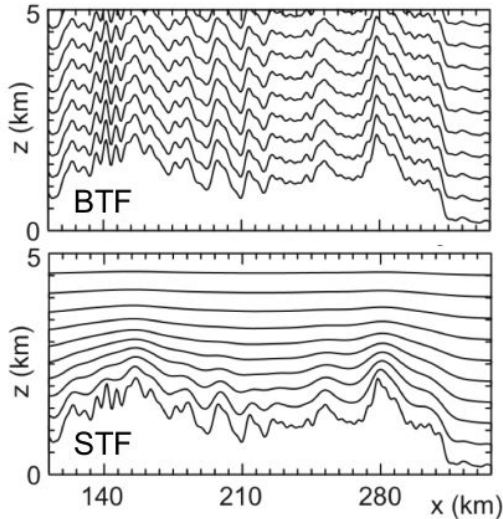


MPAS cross section through the Himalayas at 28 degrees N latitude for a 15 km (mean cell-center spacing) uniform mesh. The model top is at 30 km. The vertical coordinate in the MPAS-Atmosphere solver allows for both the traditional terrain following coordinate (right) and a general hybrid coordinate (left). From <https://mpas-dev.github.io/>.

Specification of terrain:

- High resolution terrain data (30 arcsec) averaged over grid-cell area
- Terrain smoothing with one pass of a 4th order Laplacian

Smoothed Terrain-Following (STF) hybrid Coordinate



$$z(x, y, \zeta) = \zeta + A(\zeta)h_s(x, y, \zeta)$$

$A(\zeta)$ Controls rate at which terrain influences are attenuated with height

$h_s(x, y, \zeta)$ Terrain influence that represents increased smoothing of the actual terrain with height

Multiple passes of simple Laplacian smoother at each ζ level:

$$h_s^{(n)} = h_s^{(n-1)} + \beta(\zeta)d^2\nabla_{\zeta}^2 h_s^{(n-1)}$$

STF progressively smooths coordinate surfaces while transitioning to a height coordinate

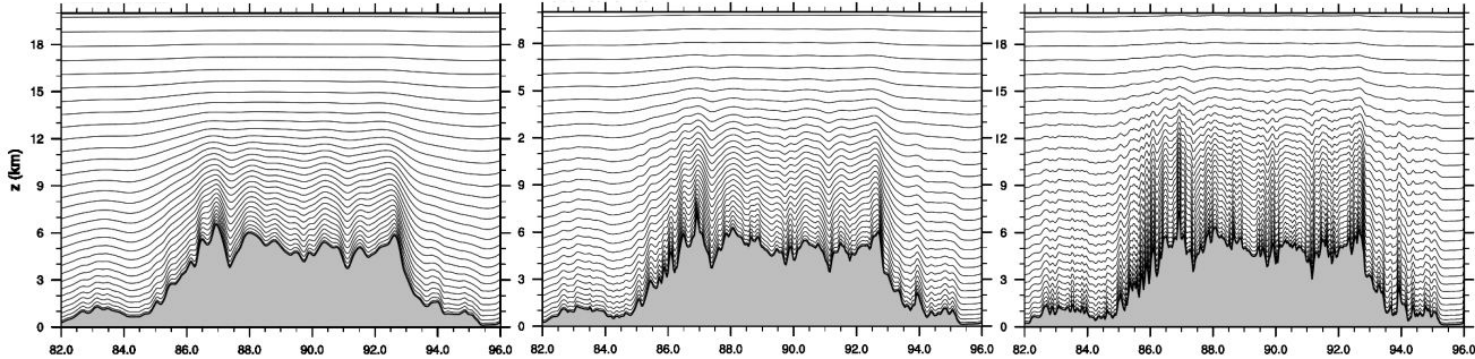
15, 7.5, & 3 km MPAS - Tibetan Plateau, 28° N

15 km grid

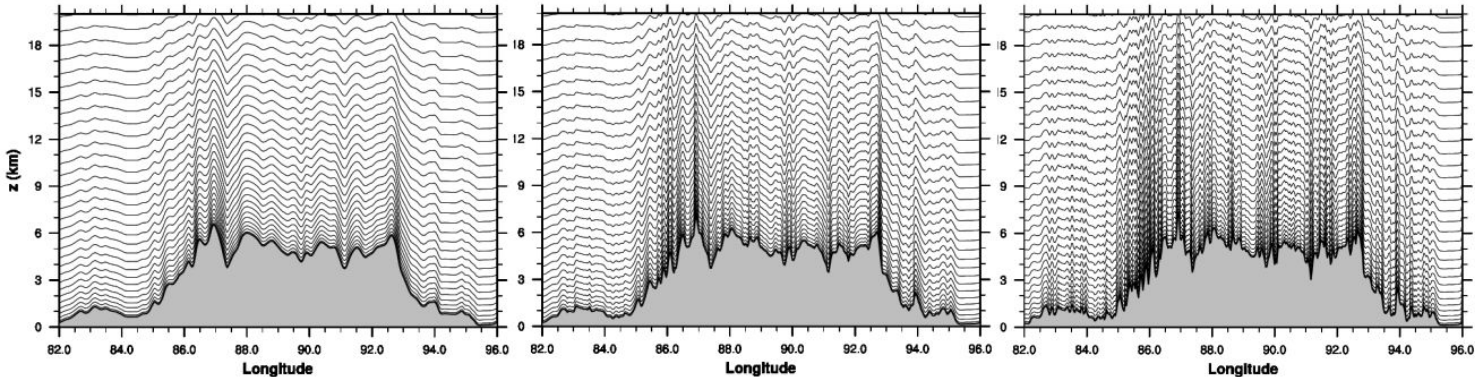
7.5 km grid

3 km grid

Smoothed hybrid terrain-following (STF) coordinate

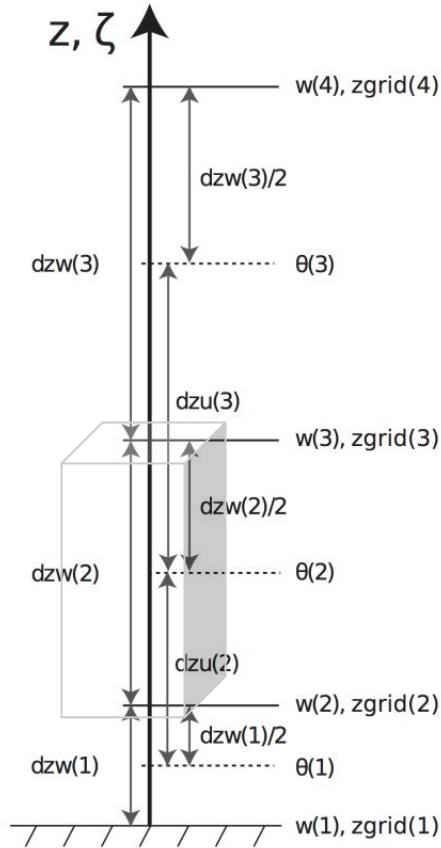


Basic terrain-following (BTF) coordinate



(Model top is at 30 km)

Vertical grid



The MPAS-Atmosphere vertical grid is also staggered:

- vertical velocities on w levels
- all other fields on Θ levels (Potential temperature)

$zgrid$ gives geometric height at w levels

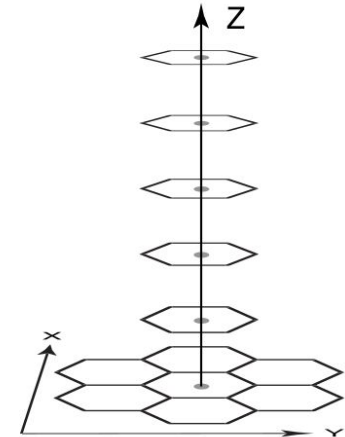
Θ levels lie at the midpoints of bracketing w levels

To vertically interpolate field F from theta levels to w levels:

$$fzp(k) = 0.5 * dzw(k) / dzu(k)$$

$$fzm(k) = 0.5 * dzw(k-1) / dzu(k)$$

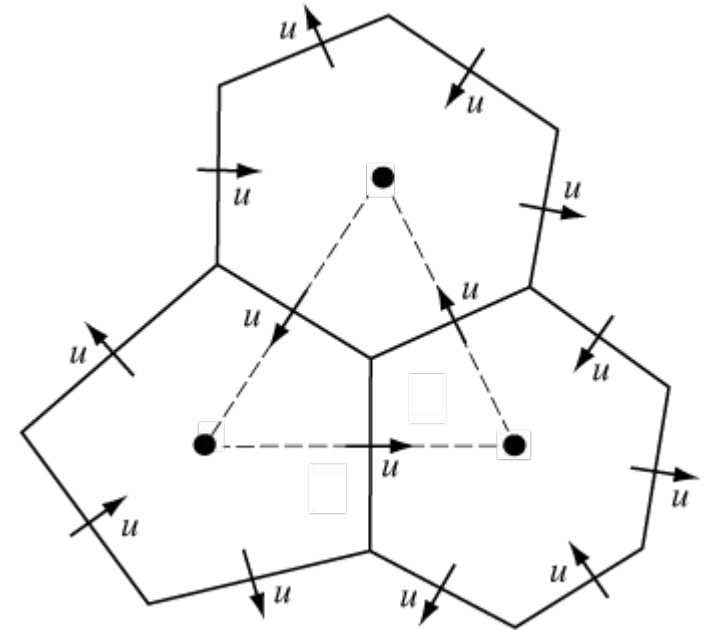
$$F_w(k) = fzm(k) * F_{\Theta}(k) + fzp(k) * F_{\Theta}(k-1)$$



Horizontal Coordinates

Staggered Voronoi Spherical Grid

C-grid staggered variables on the horizontal Voronoi mesh. Normal velocities are defined on the cell faces and all other scalar variables are defined at the cell centers. Vertical vorticity is defined at the cell vertices.



Arbitrary cell shapes (Voronoi polygons)

Nonhydrostatic formulation

Equations

- Prognostic equations for coupled variables.
- Generalized height coordinate.
- Horizontally vector invariant eqn set.
- Continuity equation for dry air mass.
- Thermodynamic equation for coupled potential temperature.

Time integration scheme

As in Advanced Research WRF -
Split-explicit Runge-Kutta (3rd order)

Variables:

$$(U, V, \Omega, \Theta, Q_j) = \tilde{\rho}_d \cdot (u, v, \dot{\eta}, \theta, q_j)$$

Vertical coordinate:

$$z = \zeta + A(\zeta) h_s(x, y, \zeta)$$

Prognostic equations:

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_{\zeta} \left(\frac{p}{\zeta_z} \right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H \\ - \mathbf{v}_H \nabla_{\zeta} \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_{\zeta} K - eW \cos \alpha_r - \frac{uW}{r_e} + \mathbf{F}_{V_H},$$

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\frac{\partial p}{\partial \zeta} + g \tilde{\rho}_m \right] - (\nabla \cdot \mathbf{v} W)_{\zeta} \\ + \frac{uU + vV}{r_e} + e(U \cos \alpha_r - V \sin \alpha_r) + F_W,$$

$$\frac{\partial \Theta_m}{\partial t} = -(\nabla \cdot \mathbf{V} \theta_m)_{\zeta} + F_{\Theta_m},$$

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -(\nabla \cdot \mathbf{V})_{\zeta},$$

$$\frac{\partial Q_j}{\partial t} = -(\nabla \cdot \mathbf{V} q_j)_{\zeta} + \rho_d S_j + F_{Q_j},$$

Diagnostics and definitions:

$$\theta_m = \theta [1 + (R_v/R_d)q_v] \quad p = p_0 \left(\frac{R_d \zeta_z \Theta_m}{p_0} \right)^{\gamma}$$

$$\frac{\rho_m}{\rho_d} = 1 + q_v + q_c + q_r + \dots$$

Prognostic Variables

$$(U, V, W, \Theta_m, Q_j) = \tilde{\rho}_d \cdot (u, v, w, \theta_m, q_j)$$

Horizontal Velocities

Vertical velocity

Moist potential temperature

Mixing ratio of water species
(vapor, cloud, rain)

$$\tilde{\rho}_d = \rho_d / \zeta_z$$

Vertical derivative of
height coord

Dry air density

Nonhydrostatic fluid-flow equations on the sphere

Horizontal Velocity Dynamics $\mathbf{V}_H = (U, V)$

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_\zeta \left(\frac{p}{\zeta_z} \right) - \frac{\partial \mathbf{z}_{HP}}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H$$

$$- \mathbf{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K$$

$$- eW \cos \alpha_r - \frac{\mathbf{v}_H W}{r_e} + \mathbf{F}_{\mathbf{V}_H}$$

Grad of pressure
 Coriolis
 Grad of Horizontal Kinetic Energy
 3D nonlinear divergence
 Sources/sinks: Physics, subgrid and filters
 Vertical and metric terms

Mass Dynamics

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -(\nabla \cdot \mathbf{V})_\zeta$$

$$\frac{\partial Q_j}{\partial t} = -(\nabla \cdot \mathbf{V} q_j)_\zeta + F_{Q_j}$$

Flux form divergence
(Ready for Finite Volumes!)

$$\frac{\partial \Theta_m}{\partial t} = -(\nabla \cdot \mathbf{V} \theta_m)_\zeta + F_{\Theta_m}$$

Summary

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_{\zeta} \left(\frac{p}{\zeta_z} \right) - \frac{\partial z_{HP}}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H$$

$$- \mathbf{v}_H \nabla_{\zeta} \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_{\zeta} K - eW \cos \alpha_r - \frac{uW}{r_e} + \mathbf{F}_{V_H},$$

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m \right] - (\nabla \cdot \mathbf{v} W)_{\zeta}$$

$$+ \frac{uU + vV}{r_e} + e(U \cos \alpha_r - V \sin \alpha_r) + F_W,$$

$$\frac{\partial \Theta_m}{\partial t} = -(\nabla \cdot \mathbf{V} \theta_m)_{\zeta} + F_{\Theta_m},$$

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -(\nabla \cdot \mathbf{V})_{\zeta},$$

$$\frac{\partial Q_j}{\partial t} = -(\nabla \cdot \mathbf{V} q_j)_{\zeta} + \rho_d S_j + F_{Q_j},$$

(1) Gradient operators

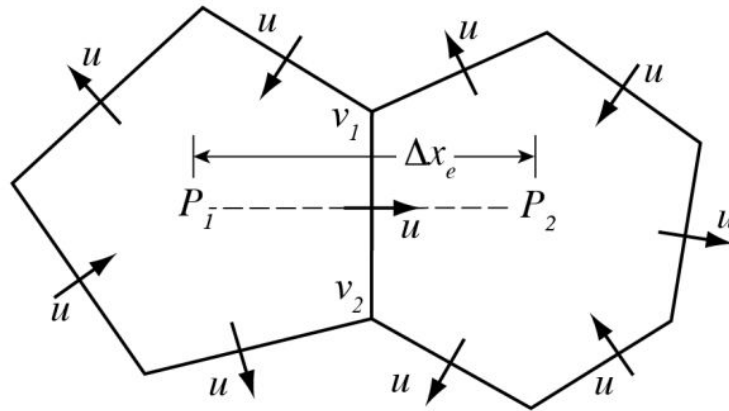
(2) Flux divergence operators

(3) Nonlinear Coriolis term

Discrete Gradient

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_\zeta \left(\frac{p}{\zeta_z} \right) - \frac{\partial z_{HP}}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H$$

$$- \mathbf{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K - eW \cos \alpha_r - \frac{uW}{r_e} + \mathbf{F}_{V_H},$$



On the Voronoi mesh, P_1P_2 is perpendicular to v_1v_2 and is bisected by v_1v_2 , hence $P_x \sim (P_2 - P_1) \Delta x_e^{-1}$ is 2nd order accurate.

Code example

MPAS-BR / src / core_atmosphere / dynamics / mpas_atm_time_integration.F

Code

Blame

6607 lines (5328 loc) · 293 KB

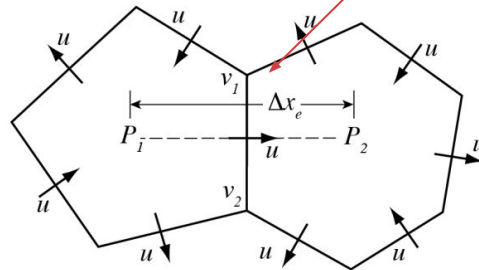


Code 55% faster with GitHub Copilot

Raw



```
4380
4381     ! horizontal pressure gradient
4382
4383     if(rk_step == 1) then
4384 !DIR$ IVDEP
4385         do k=1,nVertLevels
4386             tend_u_euler(k,iEdge) = - cqu(k,iEdge)*( (pp(k,cell2)-pp(k,cell1))*invDcEdge(iEdge)/(.5*(zz(k,cell2)+zz(k,cell1))) &
4387                 -0.5*zxu(k,iEdge)*(dpdz(k,cell1)+dpdz(k,cell2)) )
4388         end do
4389     end if
4390
```

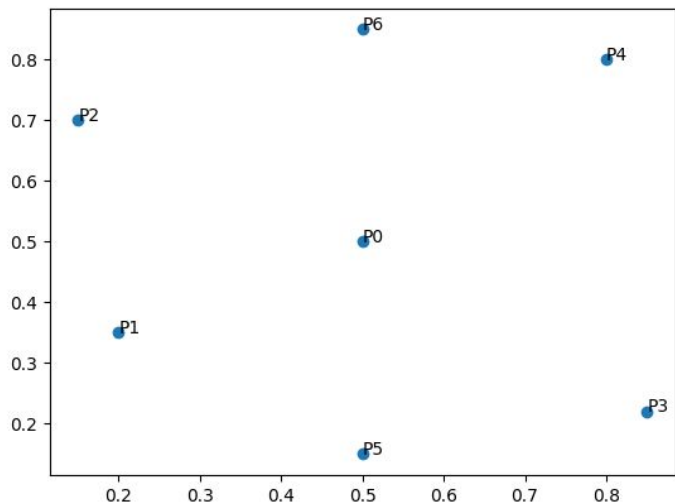


Summary

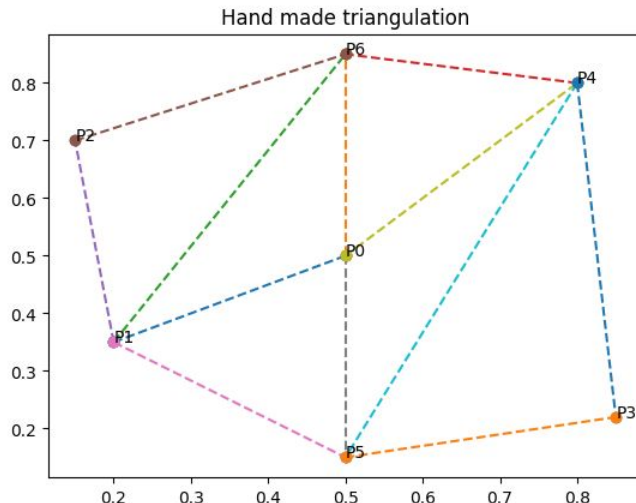
- Historical background
- Basic concepts of MONAN/MPAS dynamics
- Unstructured grids theory
- MONAN computational workflow

Grid basic concepts: Triangulation

Set of points:



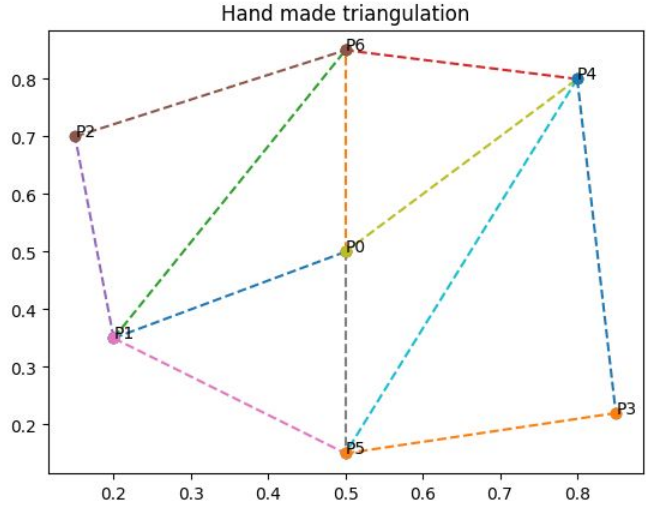
Triangulation:



Is this a good way to connect these points?

Grid basic concepts: Delaunay Triangulation

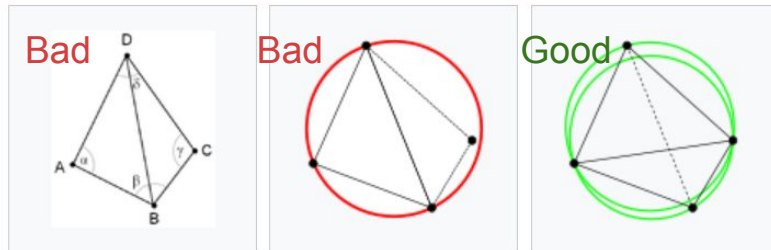
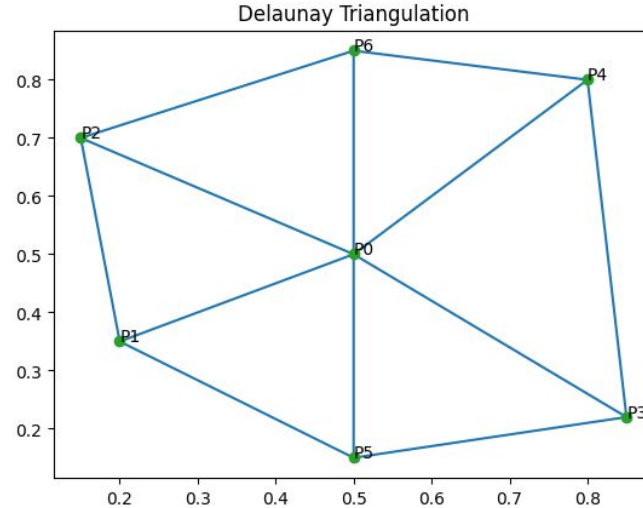
Triangulation:



Better!



Delaunay Triangulation:

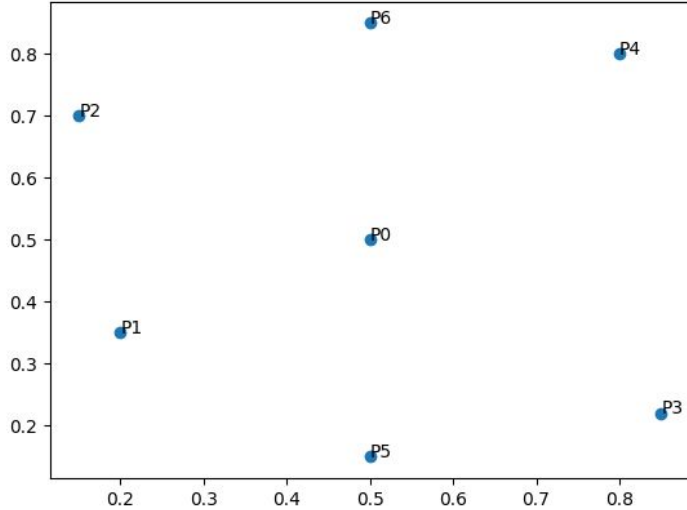


Delaunay: form triangles whose circumcircles do not contain any of the points!

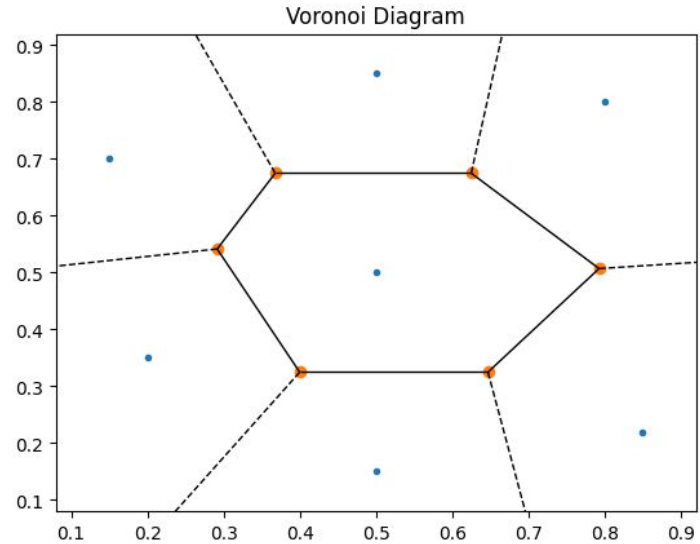
https://en.wikipedia.org/wiki/Delaunay_triangulation

Voronoi Diagram

Set of points:



Voronoi Diagram:



Voronoi cells: Each cell contains all the points in space that are nearest to that cell's single associated point

Voronoi In Nature



The Children's Museum of Indianapolis



Luca Galuzzi - www.galuzzi.it



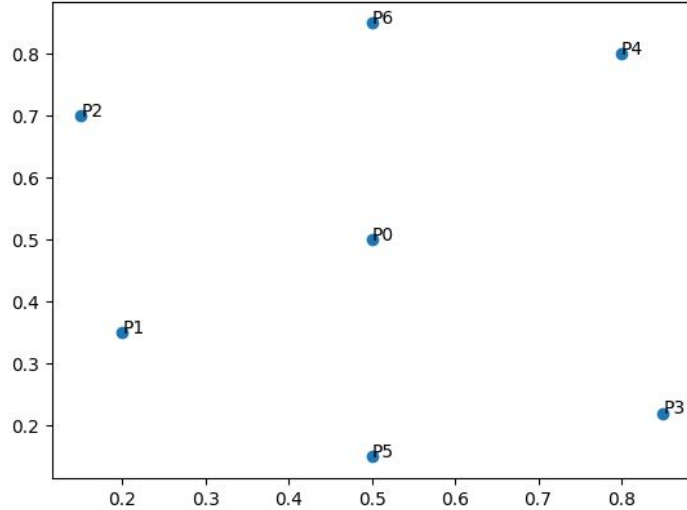
Hannes Grobe



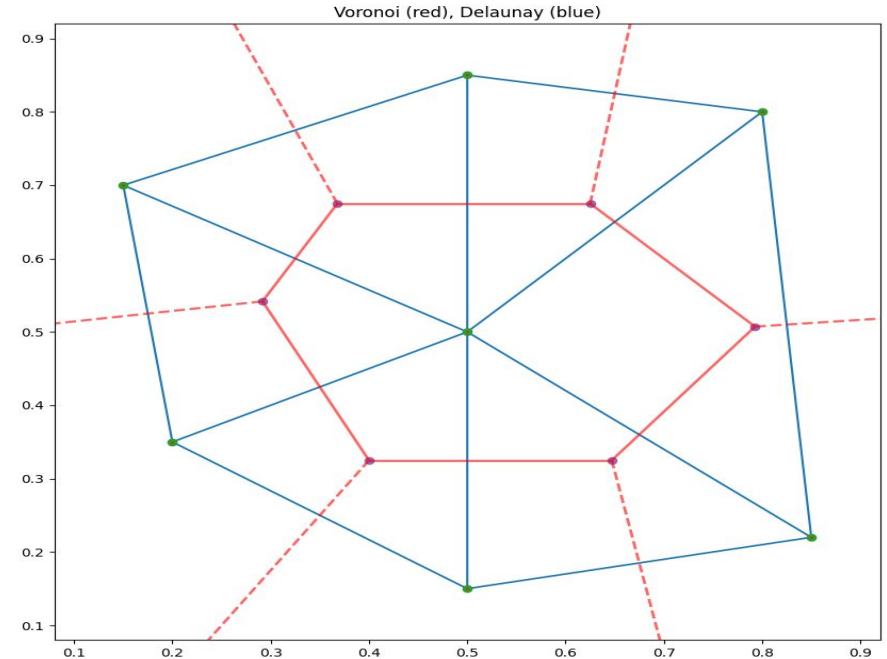
<https://www.scientificamerican.com/blog/observations/voronoi-tessellations-and-scutoids-are-everywhere/>

Delaunay-Voronoi Grid

Set of points:

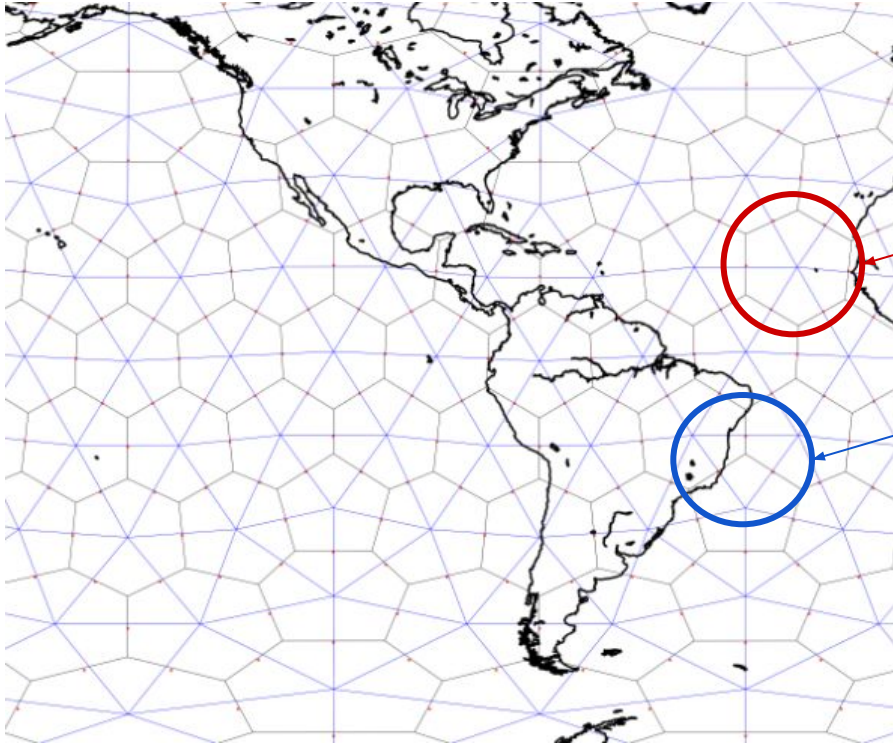


Delaunay-Voronoi Grid:



Try it out here in Colab: <https://drive.google.com/file/d/1Bc3jMxor4e7NC7GKJMnpIIZEKPBnAEHws/view?usp=sharing>

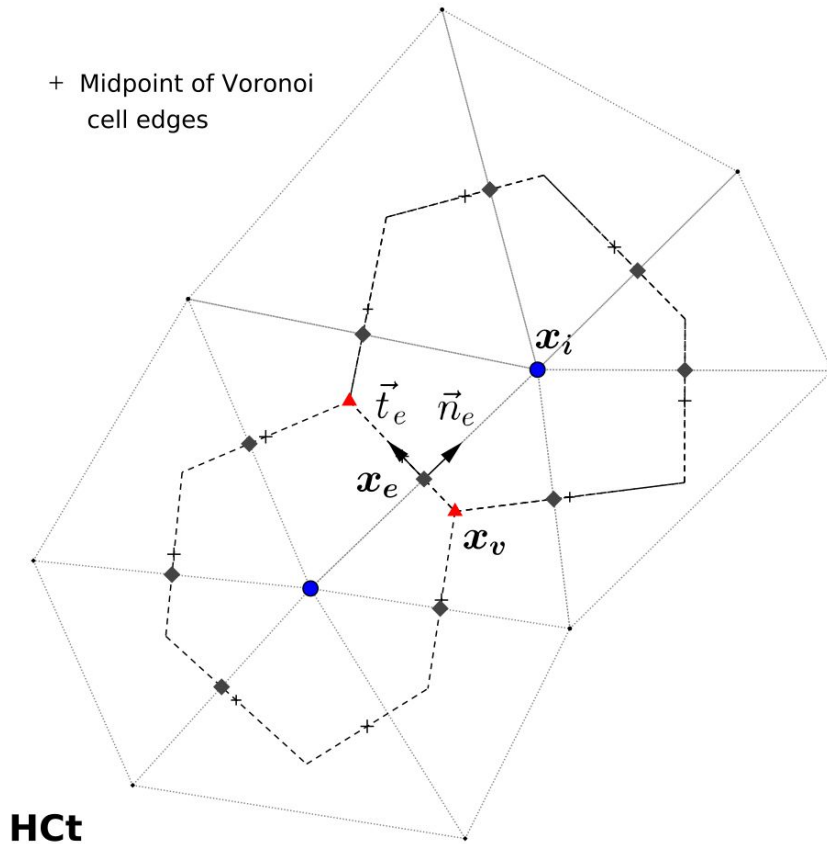
On the sphere



Primal Voronoi Cells
(Hexagons/Pentagons)

Dual Delaunay triangular grid

MPAS Grid Structures



Voronoi/Delaunay grid

Primal Grid (Voronoi):

- Cells Nodes (Voronoi generators)
- Cell Vertices (Triangle circumcenters)
- Cell Edges (connects triangle circumcenters)

Dual Grid (Delaunay):

- Triangles circumcenters
- Triangle vertices (Cell nodes)
- Triangle Edges (connects cell nodes)

Dual and Primal Edges:

- Same indexing
- Orthogonal
- Reference tangent and normal vectors

Data structure

MPAS grid files and coding structure

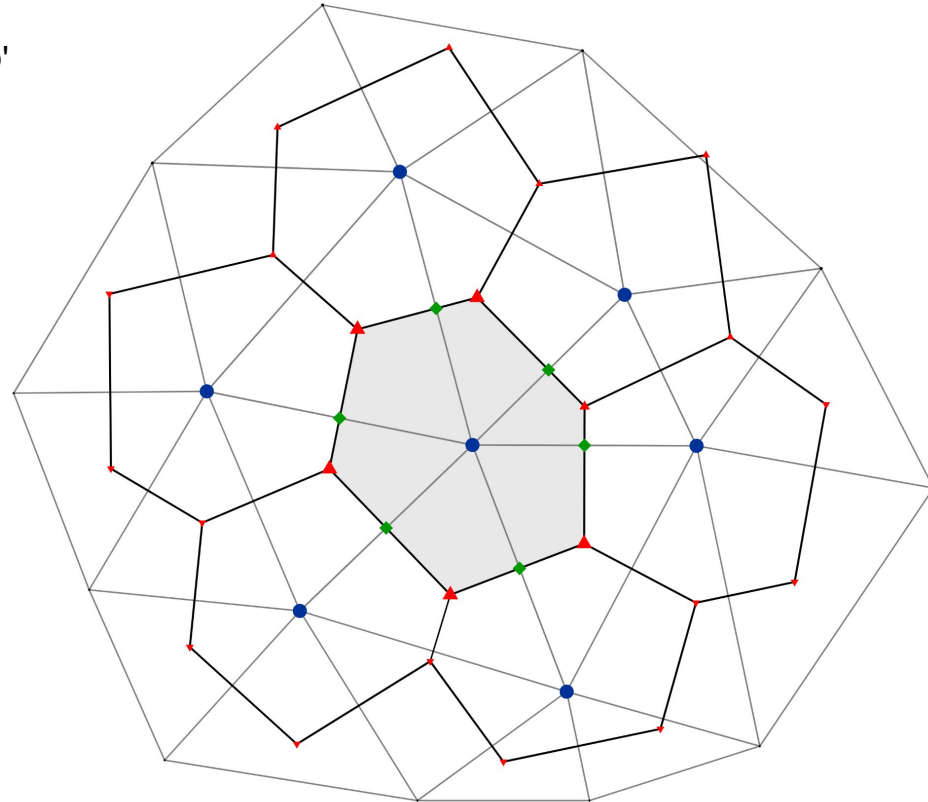
Geometric Point Locations

-Cell: 'latCell', 'lonCell', 'xCeIl', 'yCell', 'zCell', 'indexToCellID'

-Edges: 'latEdge', 'lonEdge', 'xEdge', 'yEdge', 'zEdge',
'indexToEdgeID'

-Vertices (Triangle circumcentres): 'latVertex', 'lonVertex',
'xVertex', 'yVertex', 'zVertex', 'indexToVertexID'

IndexToXXXX : Converts local indexing to global indexing
(to allow parallelism, the grid is split into blocks)



Data structure

MPAS grid files and coding structure

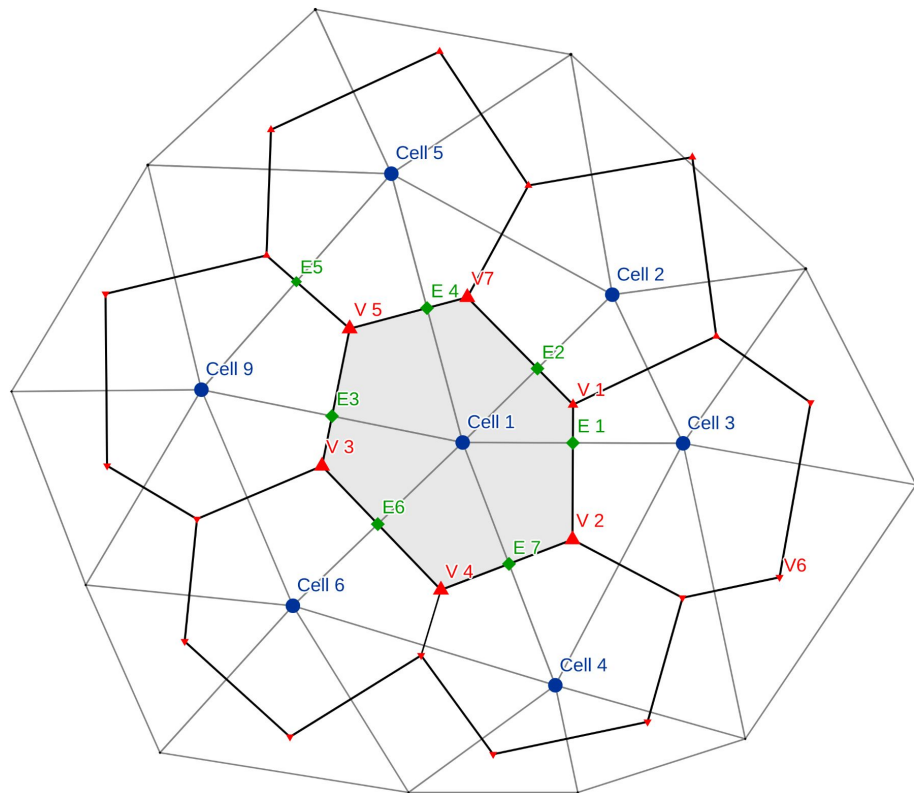
Connections:

-Quantities: 'nEdgesOnCell'

-Indexing: 'cellsOnCell', 'edgesOnCell', 'verticesOnCell',
'edgesOnEdge', 'cellsOnEdge', 'verticesOnEdge',
'cellsOnVertex', 'edgesOnVertex'

Example:

- $nEdgesOnCell(1) = 6$
- $cellsOnCell(1) = (3, 2, 5, 9, 6, 4)$
- $edgesOnCell(1) = (1, 2, 4, 3, 6, 7)$
- $verticesOnCell(1) = (1, 7, 5, 3, 4, 2)$
- $cellsOnEdge(1) = (1, 3)$
- $verticesOnEdge(1) = (1, 2)$
- $cellsOnVertex(1) = (1, 3, 2)$
- $edgesOnVertex(5) = (4, 5, 3)$

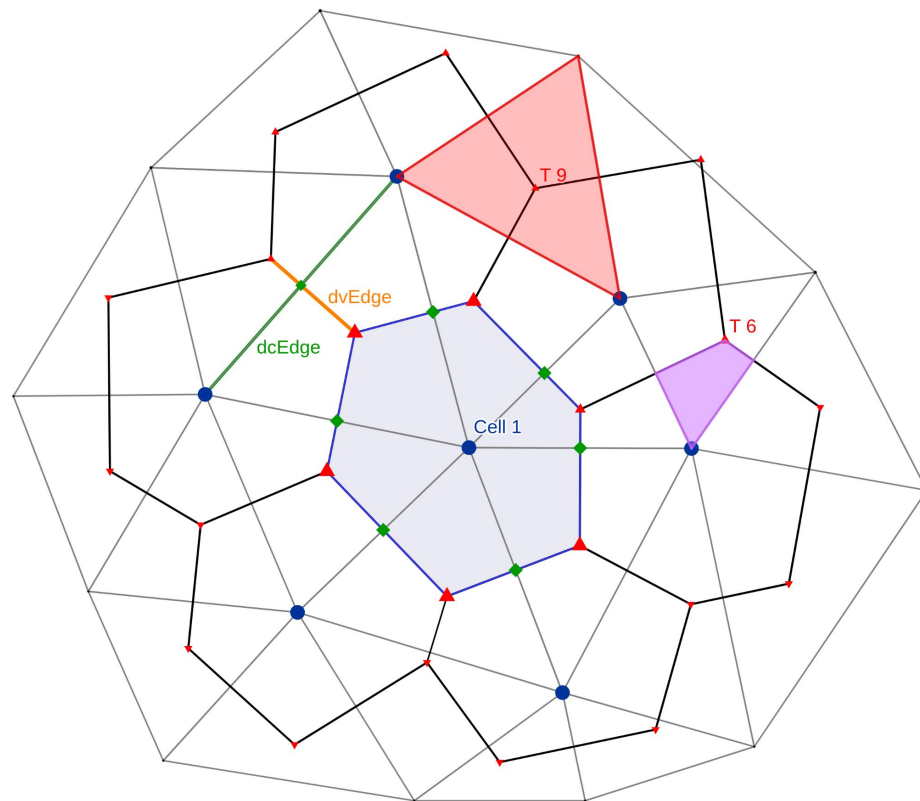


Data structure

MPAS grid files and coding structure

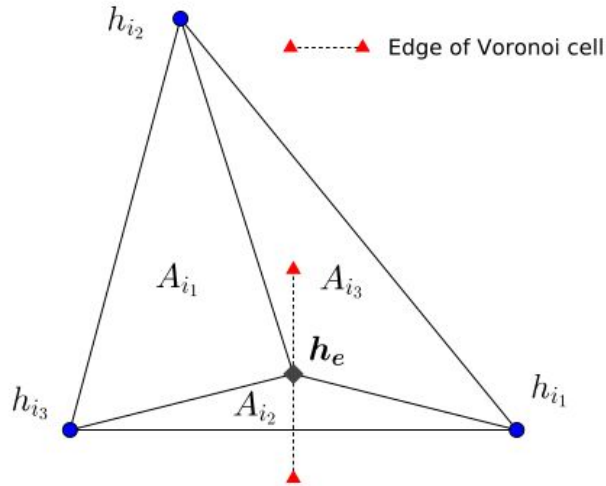
Properties:

- Area: 'areaCell', 'areaTriangle', 'kiteAreasOnVertex'
- Length: 'dcEdge', 'dvEdge'
- Quality: 'cellQuality', 'gridSpacing', 'triangleQuality', 'triangleAngleQuality', 'obtuseTriangle', 'meshDensity'
- Others: 'angleEdge', 'weightsOnEdge'

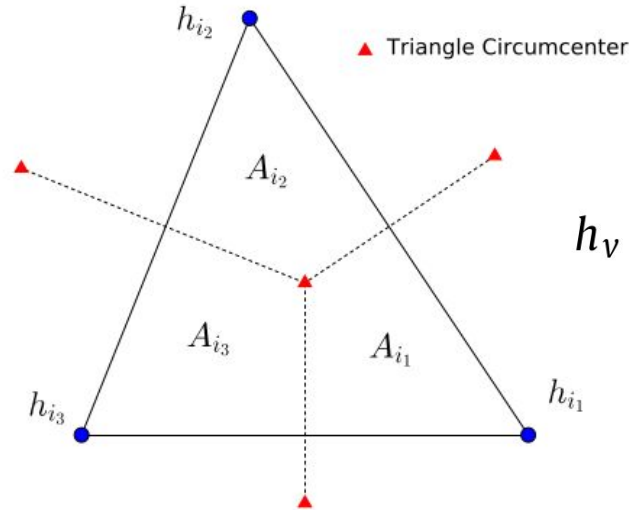


Scalar Data interpolation

From triangle vertices (cell nodes) to triangle centres:



Barycentric Coordinate Areas



Dual/Primal Intersection Areas

$$h_v = \frac{1}{A_k} \sum_i a_{iv} h_i,$$

Used in MPAS
(Exact for constant fields)

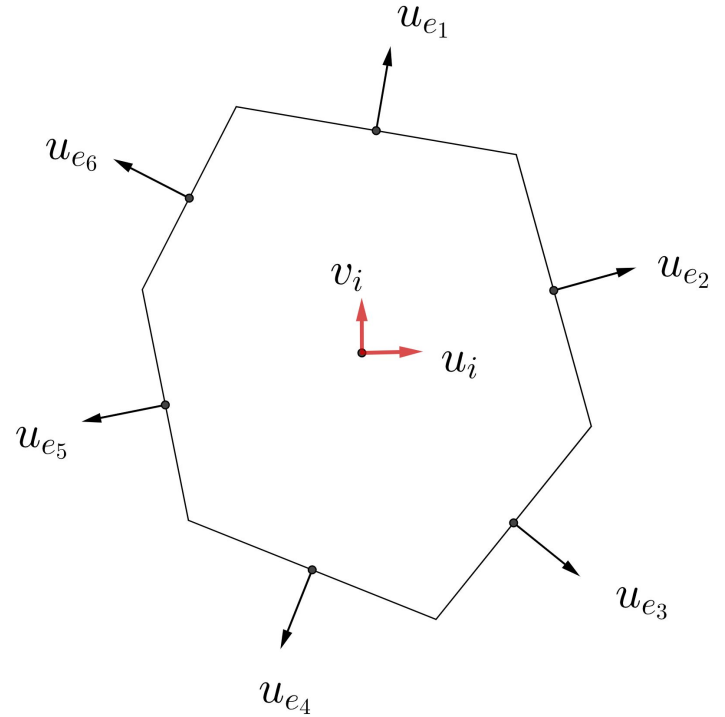
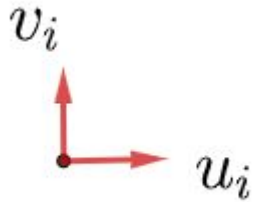
$$h_e = \sum_i \lambda_i(x_e) h_i, \quad \lambda_i(x_e) = \frac{a_i(x_e)}{A_k},$$

Barycentric: Exact for linear fields

Vector Reconstructions

Given normal edge velocities: u_e

How to reconstruct zonal/meridional velocities at cell centre?

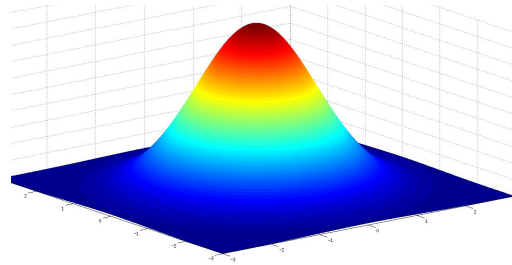


Vector Reconstruction

MPAS uses Radial Basis Functions (RBF)

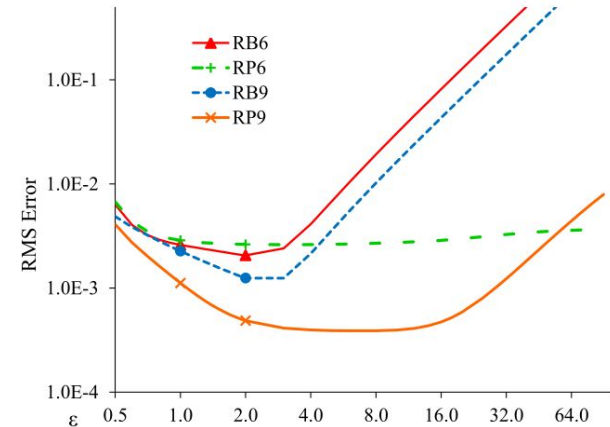
$$\vec{u}(\vec{x}) = \sum_{i=1}^k \lambda_i \phi_i(\vec{x}) \vec{n}_i,$$

$$\phi(r) = e^{-(\epsilon r)^2}$$



Solves a linear system:

$$\sum_{i=1}^k \lambda_i \phi_i(\vec{x}_j) \vec{n}_i \cdot \vec{n}_j = u_j$$



See: Peixoto, PS and Barros, SRM, 2014 : On vector field reconstructions for semi-Lagrangian transport methods on geodesic staggered grids (Journal of Computational Physics) – [PDE](#), [DOI](#)

Divergence

Finite Volume based

Wiki

$$\underbrace{\int \dots \int_U}_{n} \nabla \cdot \mathbf{F} dV = \underbrace{\oint \dots \oint_{\partial U}}_{n-1} \mathbf{F} \cdot \mathbf{n} dS$$

This equation is also known as the divergence theorem.

Key concepts:

- Point value divergence is approximated by cell integral (average): $\nabla \cdot (\vec{v}h) \approx \frac{1}{\|\Omega\|} \int_{\Omega} \nabla \cdot (\vec{v}h) dA$
- Divergence Theorem - from area integral to edge integral:

$$\frac{1}{\|\Omega\|} \int_{\Omega} \nabla \cdot (\vec{v}h) dA = \frac{1}{\|\Omega\|} \int_{\partial\Omega} h\vec{v} \cdot \vec{n} dl = \frac{1}{\|\Omega\|} \sum_e \int_{\gamma_e} h\vec{v} \cdot \vec{n} dl$$

- Approximate each edge integral with midpoint rule:

$$\frac{1}{\|\Omega\|} \sum_e \int_{\gamma_e} h\vec{v} \cdot \vec{n} dl = \frac{1}{\|\Omega\|} \sum_e h_e u_e n_{ei} l_e$$

- Interpolate depth to edges:

$$h_e = (h_i + h_j)/2$$

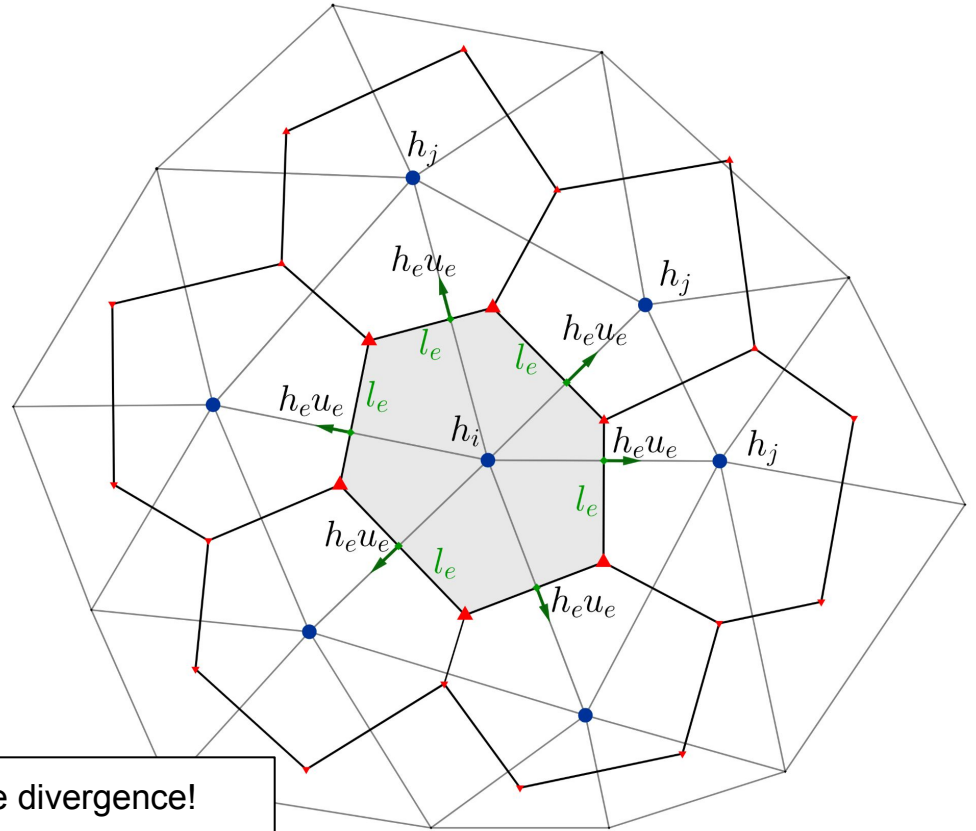
- Normal direction correction: $n_{ei} = \pm 1$

Divergence

$$D_i = \frac{1}{A_i} \sum_e h_e u_e n_{ei} l_e$$

Cell Area: A_i

Cell edge length: l_e



>> If the cell is a rectangle, we recover the 2D discrete divergence!

Divergence in MPAS

$$D_i = \frac{1}{A_i} \sum_e h_e u_e n_{ei} l_e$$

n_{ei}

l_e

```
4728
4729   do iCell=cellStart,cellEnd
4730     h_divergence(1:nVertLevels,iCell) = 0.0
4731     do i=1,nEdgesOnCell(iCell)
4732       iEdge = edgesOnCell(i,iCell)
4733       edge_sign = edgesOnCell_sign(i,iCell) * dvEdge(iEdge)
4734 !DIR$ IVDEP
4735       do k=1,nVertLevels
4736         h_divergence(k,iCell) = h_divergence(k,iCell) + edge_sign * ru(k,iEdge)
4737       end do
4738     end do
4739   end do
```

$h_e u_e$

\sum_e

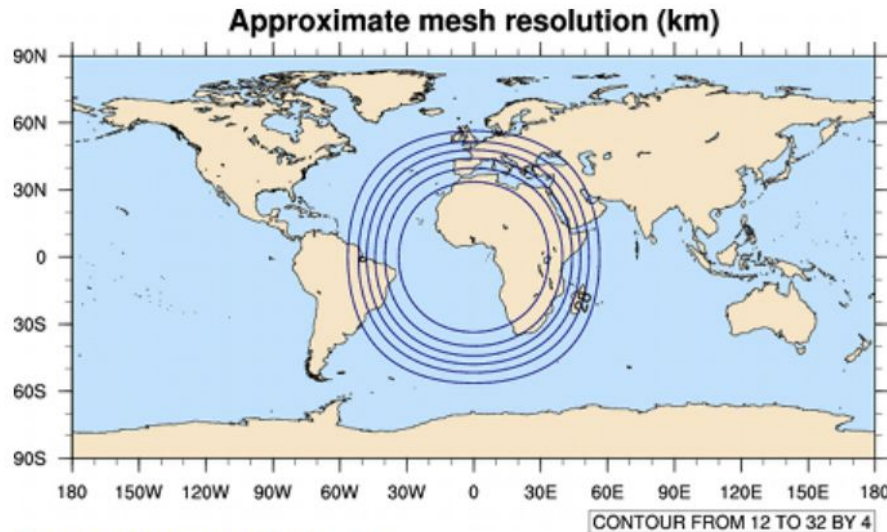
```
4743   do iCell = cellStart,cellEnd
4744     r = invAreaCell(iCell)
4745     do k = 1,nVertLevels
4746       h_divergence(k,iCell) = h_divergence(k,iCell) * r
4747     end do
4748   end do
```

$\frac{1}{A_i}$

MPAS Official Grids

https://mpas-dev.github.io/atmosphere/atmosphere_meshes.html

46-km – 12-km mesh



[Download the mesh](#) (592 MB)

Pros

- Well-tested and optimized grids

Cons

- Slow grid generation (days/weeks) ⇒ Only few grids available

120-km mesh (40962 horizontal grid cells)

[Download the 120-km mesh](#) (25.7 MB)

[Download the 120-km static file](#) (16.2 MB)

60-km mesh (163842 horizontal grid cells)

[Download the 60-km mesh](#) (106 MB)

[Download the 60-km static file](#) (69.6 MB)

48-km mesh (256002 horizontal grid cells)

[Download the 48-km mesh](#) (182 MB)

[Download the 48-km static file](#) (174 MB)

30-km mesh (655362 horizontal grid cells)

[Download the 30-km mesh](#) (436 MB)

[Download the 30-km static file](#) (296 MB)

Jigsaw Grids

Personalize your grids for MPAS

Jigsaw software: https://mpas-dev.github.io/MPAS-Tools/0.26.0/mesh_creation.html
<https://github.com/dengwirda/jigsaw-python>

Pros

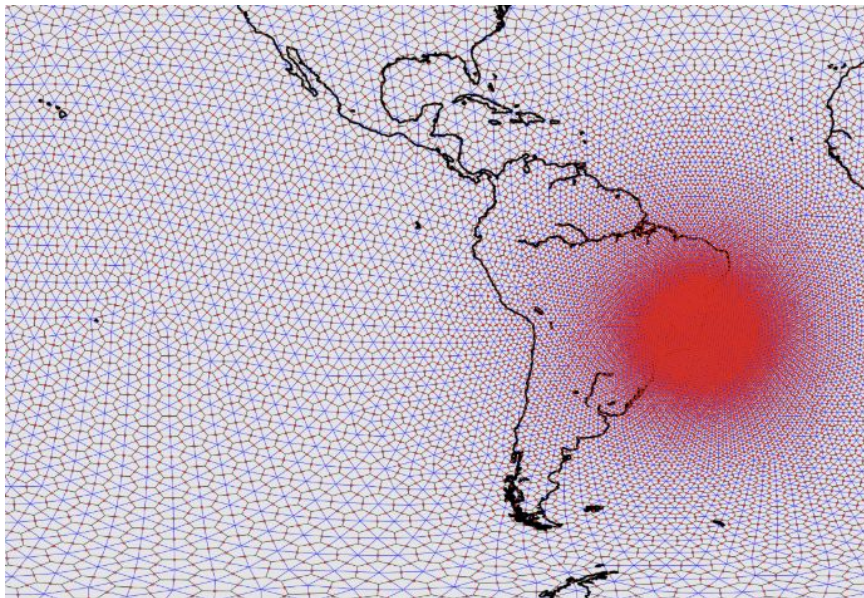
- Flexibility to generate personalized grids
- Fast algorithm to generate grids (seconds/minutes)

Cons

- Generated grids may not be as well optimized as the ones provided by NCAR

Try at home:

https://github.com/CGFD-USP/MPAS-References/blob/master/WMO-Workshop2025/JigsawGrids_tutorial.ipynb

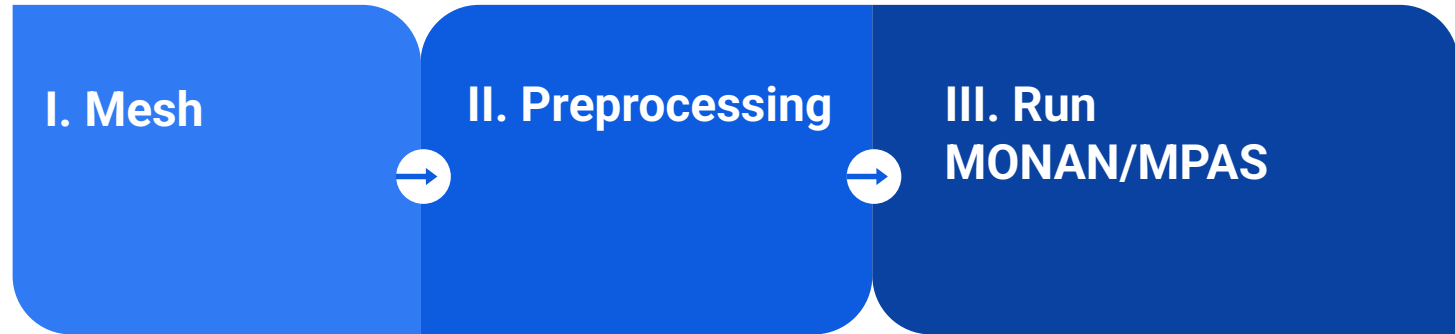


Summary

- Historical background
- Basic concepts of MONAN/MPAS dynamics
- Unstructured grids theory
- MONAN computational workflow

Tutorials

MONAN computational workflow



- a) Download official mesh
- b) Generate your own mesh

- a) Idealized simulation:
Generate artificial initial conditions

- b) Real-case simulation:
Preprocess real data for “static fields” (albedo, vegetation characteristics, etc.) and initial conditions (e.g. ERA5 or GFS)

- a) Idealized simulation:
Run dynamical core only

- b) Real-case simulation:
Run dynamical core + physical parametrizations

Tutorial 1: Mesh generation

I. Mesh

- a) Download official mesh
- b) Generate your own mesh

Tutorial 1 - Create your own meshes:

https://github.com/CGFD-USP/scripts_CD-CT/blob/feature/scripts-849-NF-idealized/docs/tutorial1-meshes.md

Tutorial 1: Mesh generation

I. Mesh

a) Download official MPAS provided meshes (few available options):

https://mpas-dev.github.io/atmosphere/atmosphere_meshes.html

b) Generate your own mesh using MONAN workflow

Personalized mesh generation tool:

- Jigsaw software

(<https://github.com/dengwirda/jigsaw-python/tree/master/tests>)

by D. Engwirda

- vtx-mpas-meshes

(<https://github.com/marta-gil/vtx-mpas-meshes>) by M. Badarji

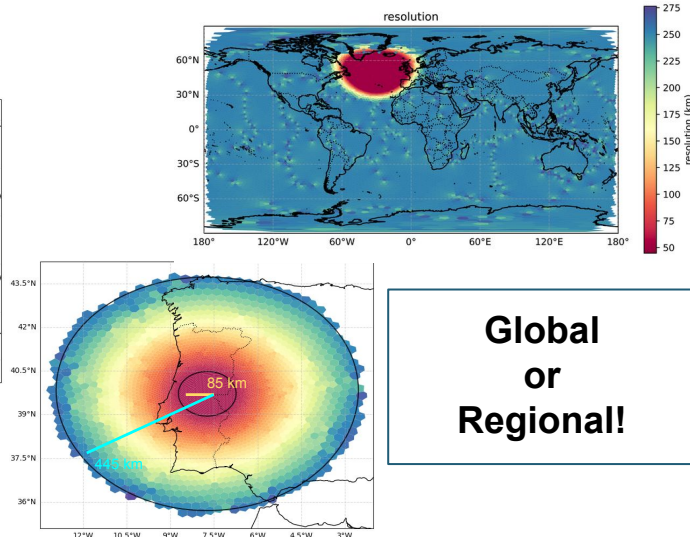
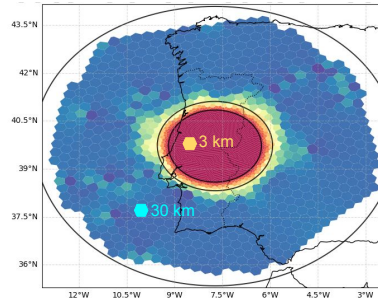
- MPAS-Tools (<https://mpas-dev.github.io/MPAS-Tools>) by

X. Asay-Davis, M. Duda et al

- MPAS-Limited-Area

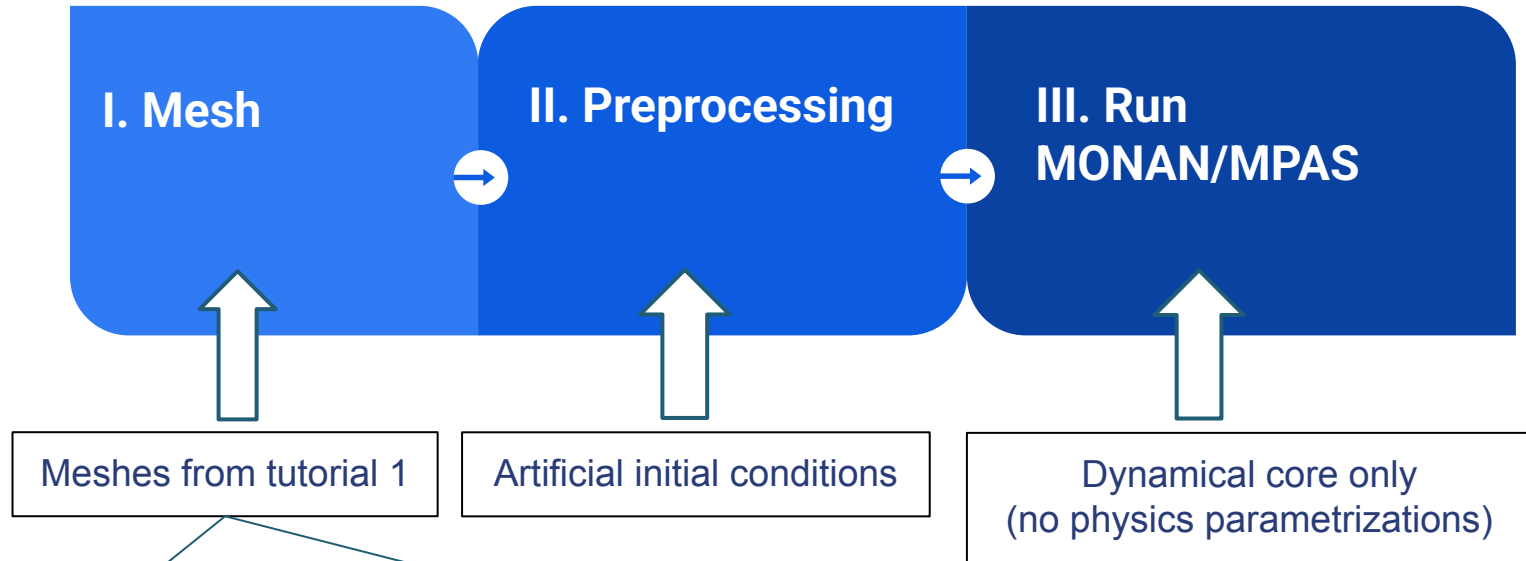
(<https://github.com/MPAS-Dev/MPAS-Limited-Area>) by

M. Duda et al



**Global
or
Regional!**

Tutorial 2: Idealized simulation



- Group 1: uniform 240 km
- Generated with jigsaw
 - Downloaded from NCAR

- Group 2: refined 48 km - 240 km
- Generated with jigsaw

Tutorial 2 - Idealized simulation:
https://github.com/CGFD-USP/scripts_CD-CT/blob/feature/scripts-849-NF-idealized/docs/tutorial2-ideal-case.md

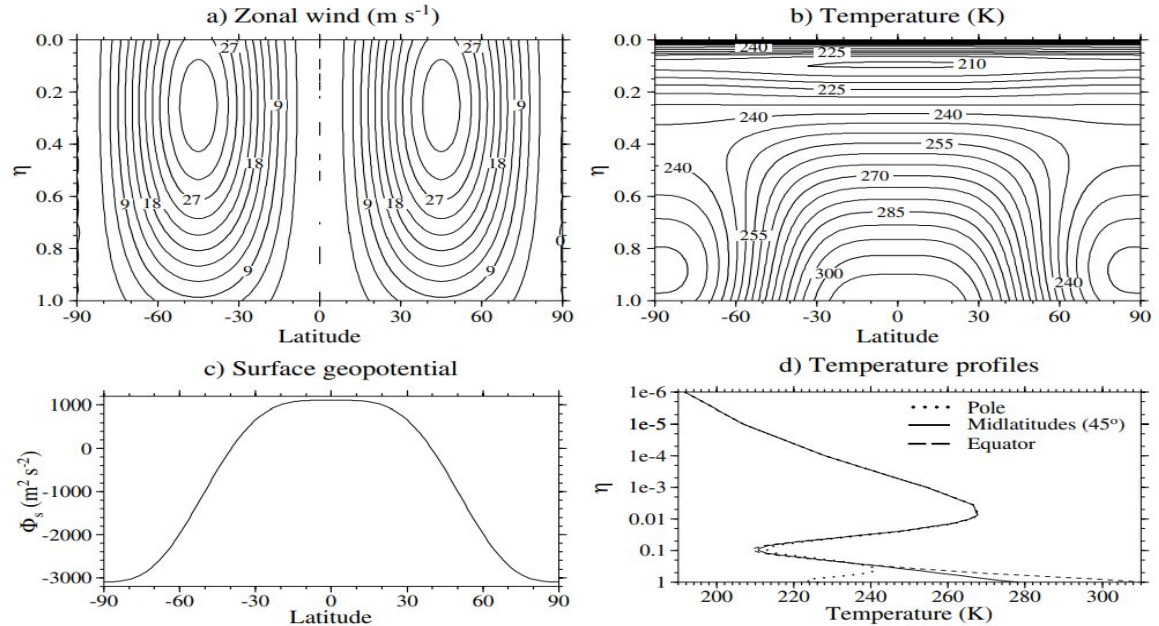
Tutorial 2: Idealized simulation

II. Preprocessing

JW Baroclinic Instability

Balanced solution!

IDEALIZED INITIAL CONDITIONS



Jablonowski, C., & Williamson, D. L. (2006). A baroclinic instability test case for atmospheric model dynamical cores. Quarterly Journal of the Royal Meteorological Society, 132(621C), 2943-2975. https://www.gfdl.noaa.gov/wp-content/uploads/files/user_files/pjp/qj_jablonowski_williamson_2006.pdf

Tutorial 2: Idealized simulation

II. Preprocessing

“init_atmosphere” code

5) Preprocessing

5.1) Edit 0.run_all.bash

We start by editing 0.run_all.bash :

```
vi 0.run_all.bash
```

```
github_link="https://github.com/monanadmin/MONAN-Model.git"  
monan_branch=release/1.4.1-rc  
convertmpas_branch=release/1.2.0  
EXP=IDEALIZED2  
YYYYMMDDHHI=2025111800  
FCST=360
```

MPAS user guide

7.1 Creating idealized ICs

There are several idealized test cases supported within the `init_atmosphere` model initialization core:

- 1 — Jablonowski and Williamson baroclinic wave, no initial perturbation ¹
- 2 — Jablonowski and Williamson baroclinic wave, with initial perturbation
- 3 — Jablonowski and Williamson baroclinic wave, with normal-mode perturbation
- 4 — squall line
- 5 — super-cell
- 6 — mountain wave

Tutorial 2: Initial conditions

II. Preprocessing

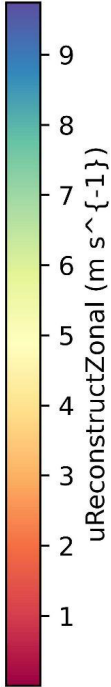
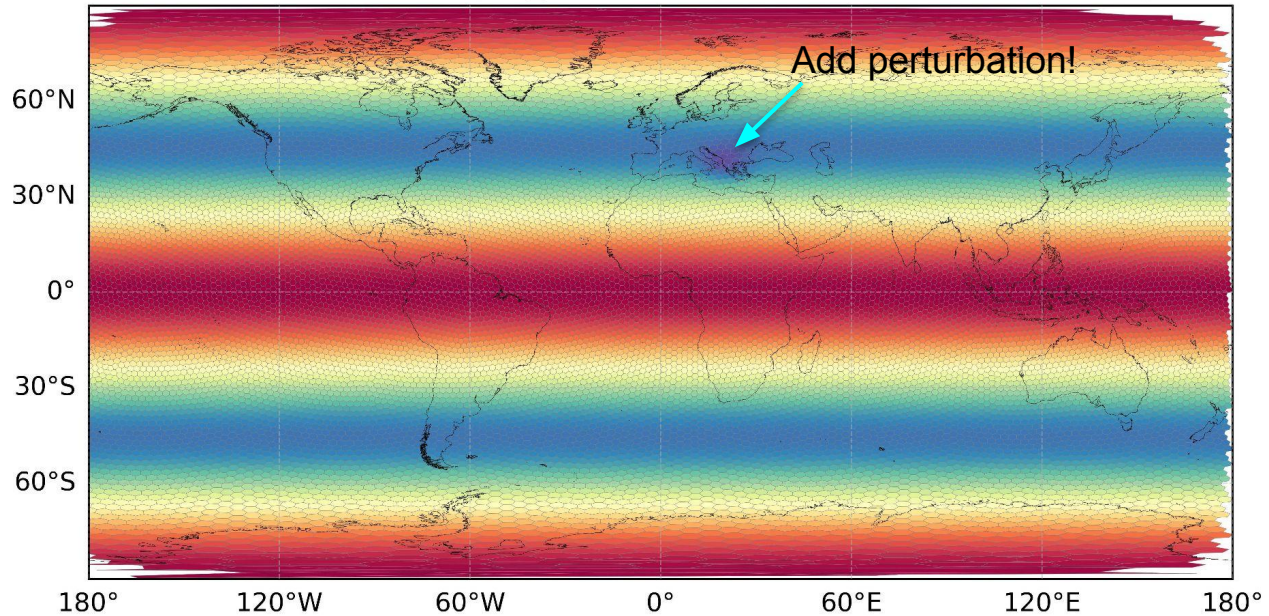
```
bash 0.run_all.bash
```



```
$MESH .init.nc
```



uReconstructZonal: output_unif200km-jw-bi.nc (6611436) Timestep=0



Tutorial 2: Running the idealized simulation

III. Run MONAN/MPAS

“atmosphere” code

6) Running the global simulation

```
vi 0.run_all.bash
```

```
# STEP 4: Executing the Model run:  
time ${SCRIPTS}/4.run_model.bash ${EXP} ${MESH} ${YYYYMMDDHH} ${FCST} ${RES}  
#exit
```

```
bash 0.run_all.bash
```

```
5 namelist atmosphere  
6  
7 &hydry_model  
8   config_dt = 720.0  
9   config_start_time = '0000-01-01_00:00:00'  
10  config_run_duration = '15_00:00:00'  
11  config_split_dynamics_transport = false  
12  config_number_of_sub_steps = 6  
13  config_dynamics_split_steps = 1  
14  config_h_mon_eddy_visc2 = 0.0  
15  config_h_mon_eddy_visc4 = 0.0  
16  config_v_mon_eddy_visc2 = 0.0  
17  config_h_theta_eddy_visc2 = 0.0  
18  config_h_theta_eddy_visc4 = 0.0  
19  config_v_theta_eddy_visc2 = 0.0  
20  config_horiz_mixing = '2d_smagorinsky'  
21  config_len_disp = 200000.  
22  config_u_vadv_order = 3  
23  config_w_vadv_order = 3  
24  config_theta_vadv_order = 3  
25  config_scalar_vadv_order = 3  
26  config_theta_adv_order = 3  
27  config_scalar_adv_order = 3  
28  config_scalar_advection = false  
29  config_positive_definite = false  
30  config_coef_3rd_order = 1.0  
31  config_monotonic = false  
32  config_apsm = 0.1  
33  config_smdiv = 0.1  
34  
35 /  
36 &damping  
37   config_zd = 22000.0  
38   config_xnutr = 0.0  
39 /  
40 &decomposition  
41   config_block_decomp_file_prefix = 'unif200km_graph.info.part.'  
42 /  
43 &restart  
44   config_do_restart = false  
45 /  
46 &printout  
47   config_print_global_minmax_vel = true  
48   config_print_global_minmax_sca = false  
49 /  
50 &physics  
51   config_physics_suite = 'none'  
52 /
```

No physics!



Tutorial 2: Results

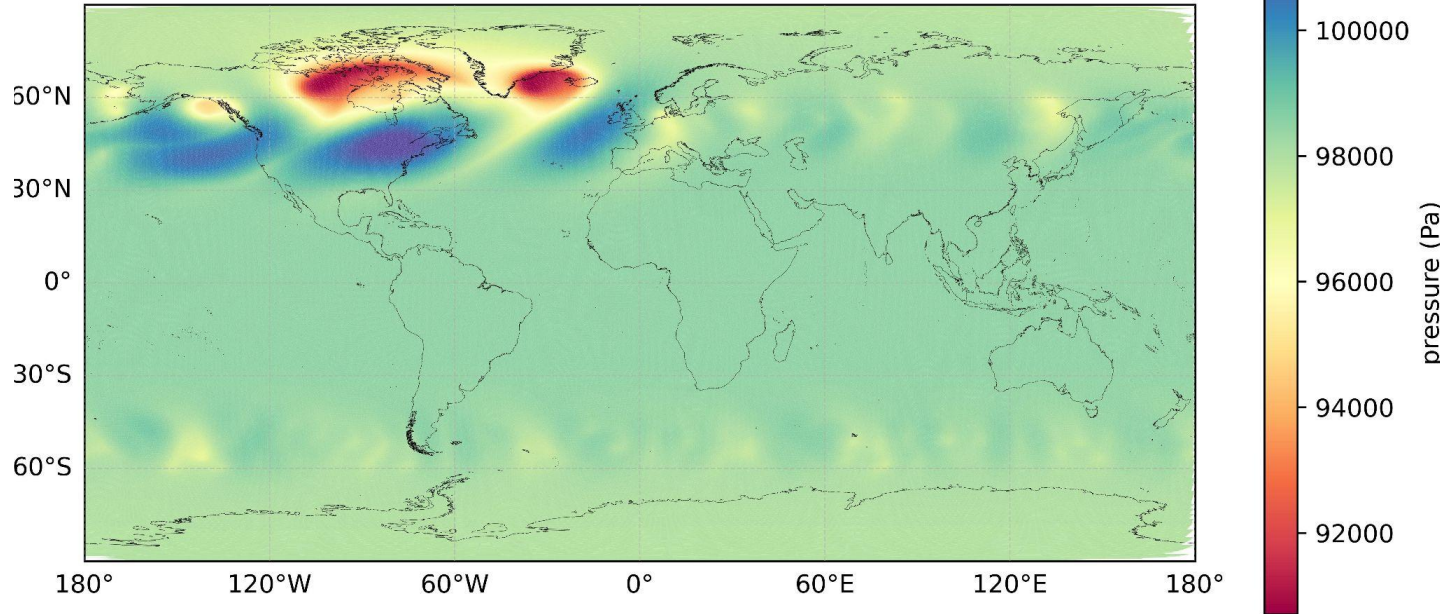
```
vi 5.run_post_on_mpas_grid.bash
```



```
# Local variables----  
## Variable to plot  
VAR=pressure
```



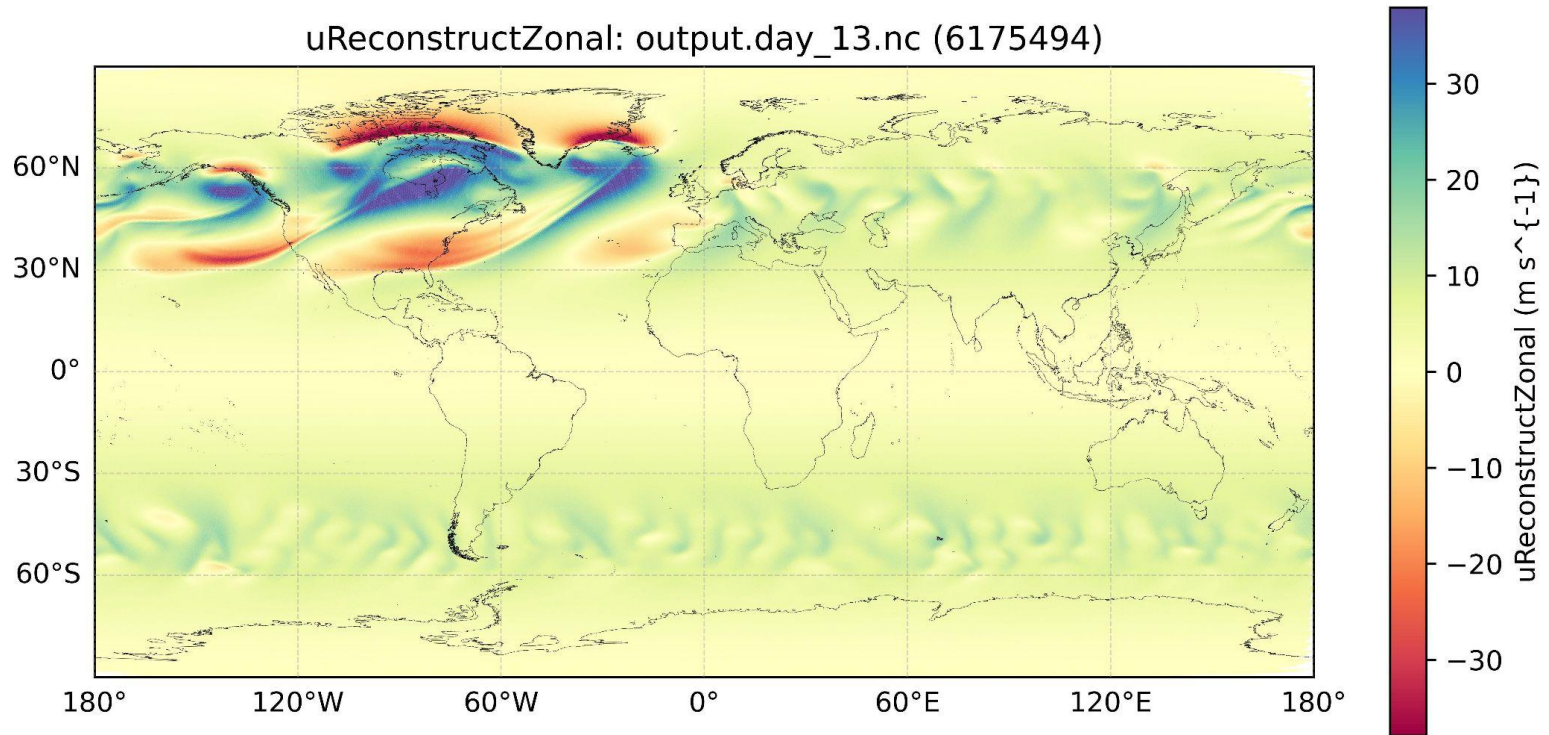
pressure: output.day_13.nc (6175494)



JW BI case with 50 km grid resolution: Pressure field at day 13

Tutorial 2: Results

JW BI case with 50 km grid resolution: Zonal wind field at day 13



Tutorial 2: Animations

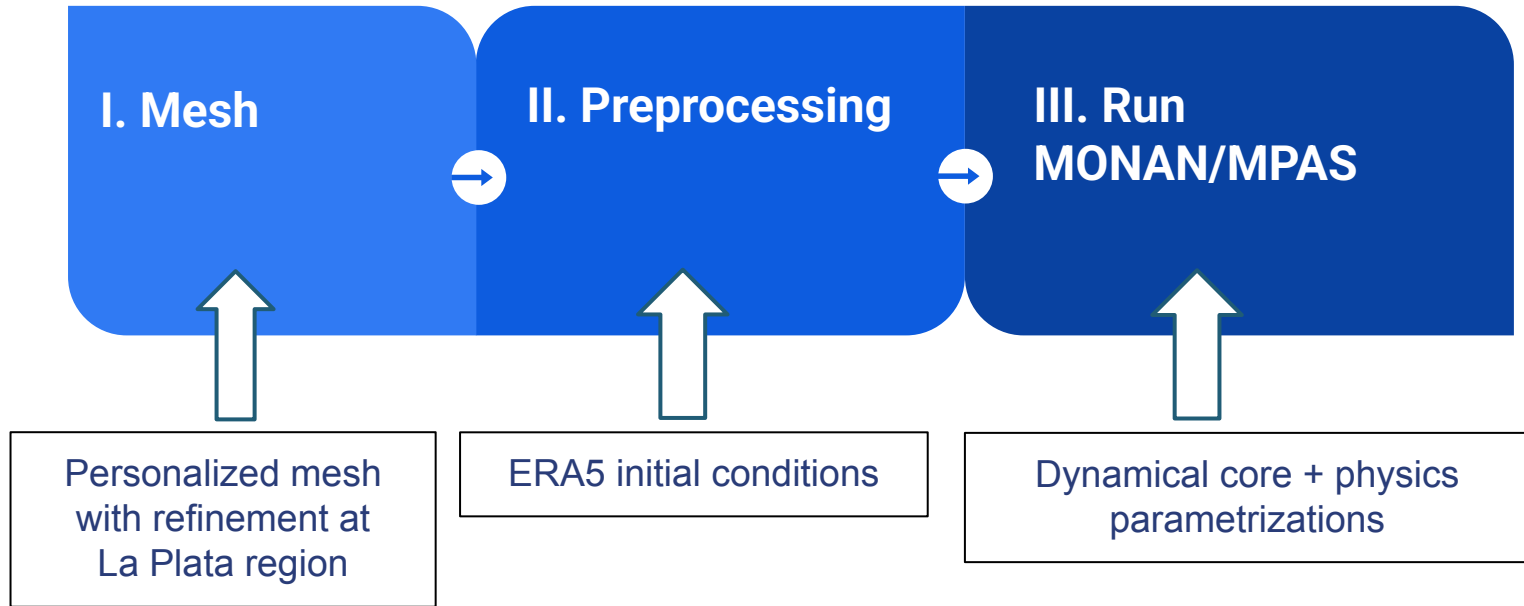
JW BI case

- 50 km grid resolution (Jigsaw)
- Level 0 (ground)

Evolution of fields:

- Vorticity
- Zonal Wind
- Meridional Wind
- Pressure

Tutorial 3: Real-case simulation



Tutorial 3 - Real-case simulation:

https://github.com/CGFD-USP/scripts_CD-CT/blob/feature/scripts-849-N-F-idealized/docs/tutorial3-real-case.md

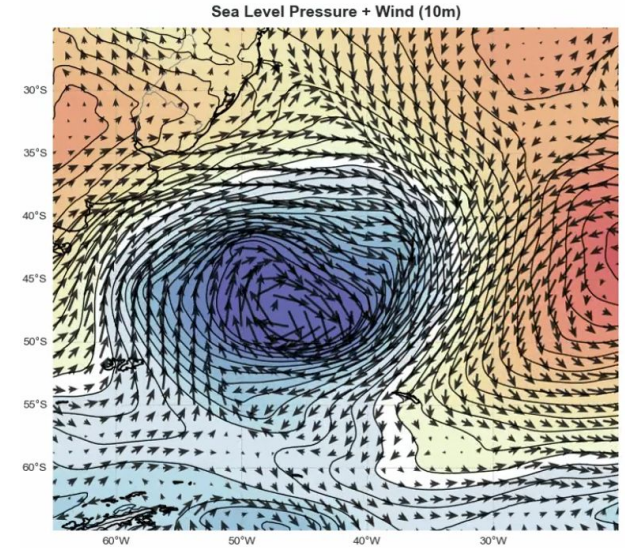
Tutorial 3: Real-case simulation

Bomb-extratropical cyclone

Presentation:

https://www.canva.com/design/DAG12r0AyUU/sZfZk8xb9T4rvqOhPoElxw/vi-ew?utm_content=DAG12r0AyUU&utm_campaign=designshare&utm_medium=link&utm_source=viewer

Dias Pinto, J. R., & Da Rocha, R. P. (2011). The energy cycle and structural evolution of cyclones over southeastern South America in three case studies. *Journal of Geophysical Research: Atmospheres*, 116(D14).



Tutorial 3: Personalized mesh generation

I. Mesh

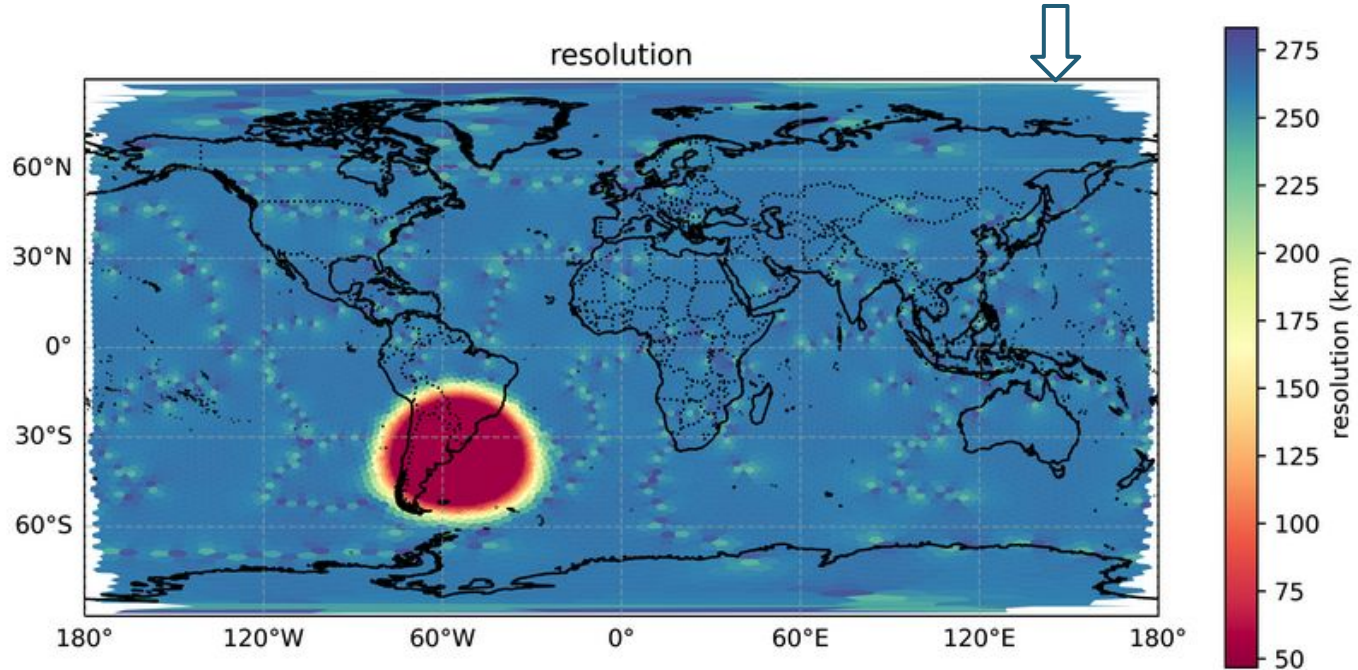
mesh_input_file.txt



2.create_mesh.bash



plot_mpas_grid.bash

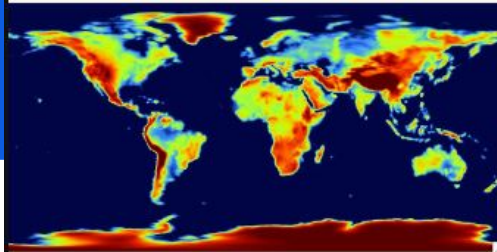


Tutorial 3: Static fields

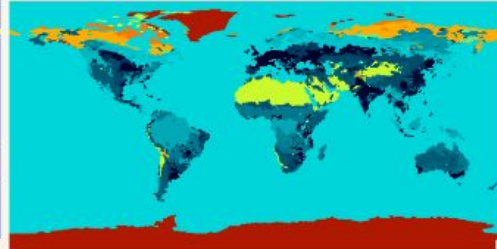
```
3.pre_processing.bash
```

Time-invariant “static” data

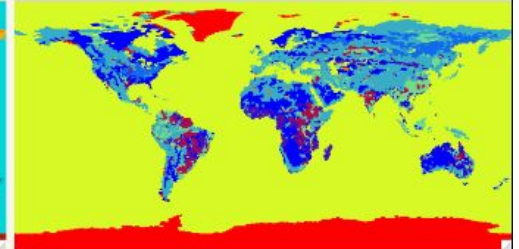
II. Preprocessing



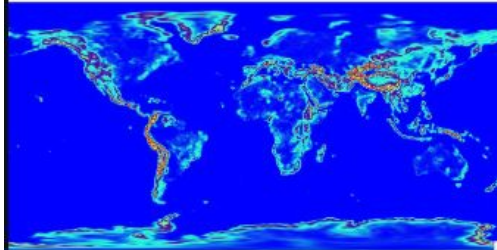
Terrain elevation



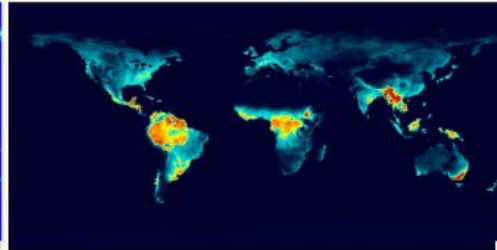
Dominant land use category



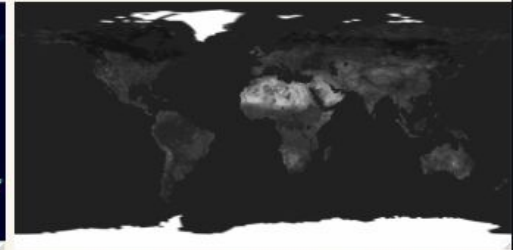
Dominant soil category



Sub-grid-scale terrain variance



*Climatological monthly
vegetation fraction*



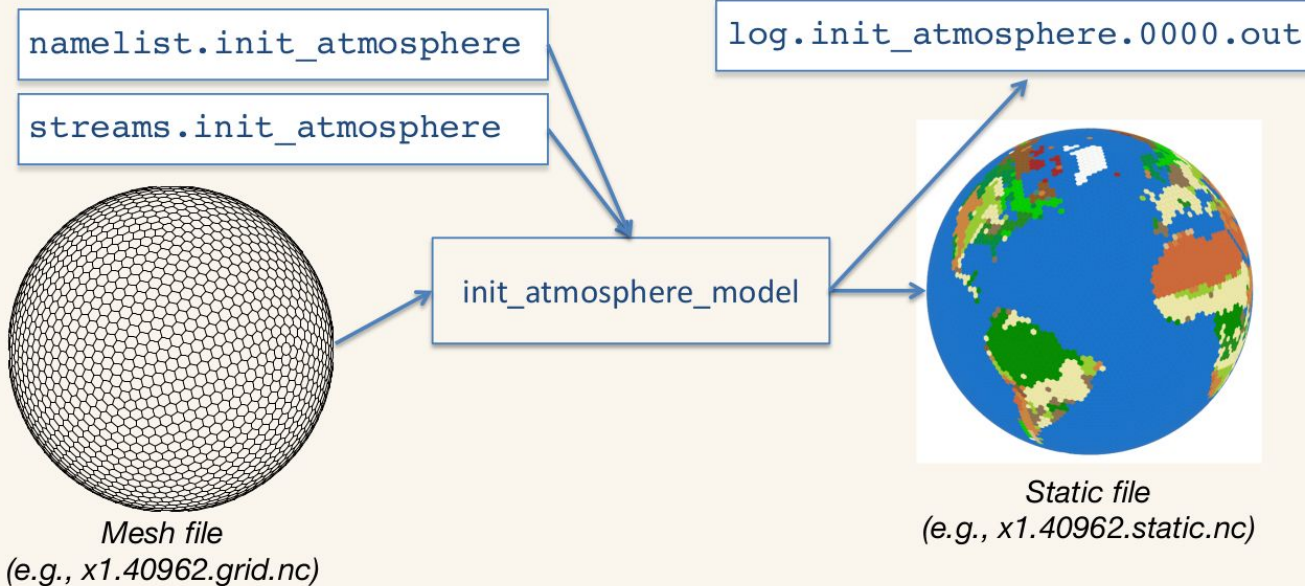
*Climatological monthly surface
albedo*

Tutorial 3: Static fields

3.pre_processing.bash

II. Preprocessing

Input and output files when producing a “static” file:

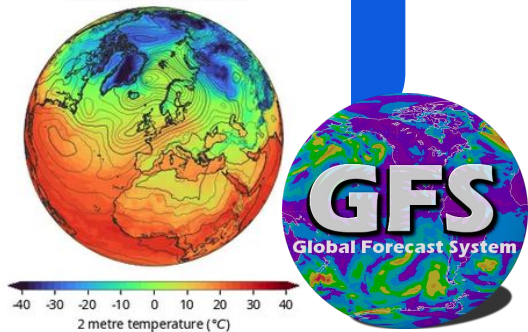


Tutorial 3: Real initial conditions

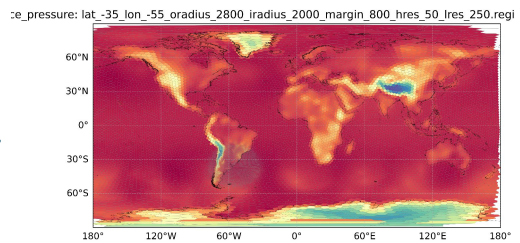
II. Preprocessing

```
3.pre_processing.bash
```

ERA5 2 metre temperature and Mean sea level pressure
1 January 2023 at 00:00 UTC



```
ERA5:2007-06-22_00
```



External meteorological data
(supports ERA5 and GFS)

Preprocessed data in simple
“intermediate” format that can be
read by MONAN

Real initial conditions on
MONAN mesh

```
vi 0.run_all.bash  
  
github_link="https://github.com/monanadmin/MONAN-Model.git"  
monan_branch=release/1.4.1-rc  
convertmpas_branch=release/1.2.0  
EXP=ERA5  
YYYYMMDDHH=2007062200
```

Tutorial 3: Running the real-case simulation

III. Run MONAN/MPAS

“atmosphere” code

6) Running the global simulation

```
vi 0.run_all.bash
```

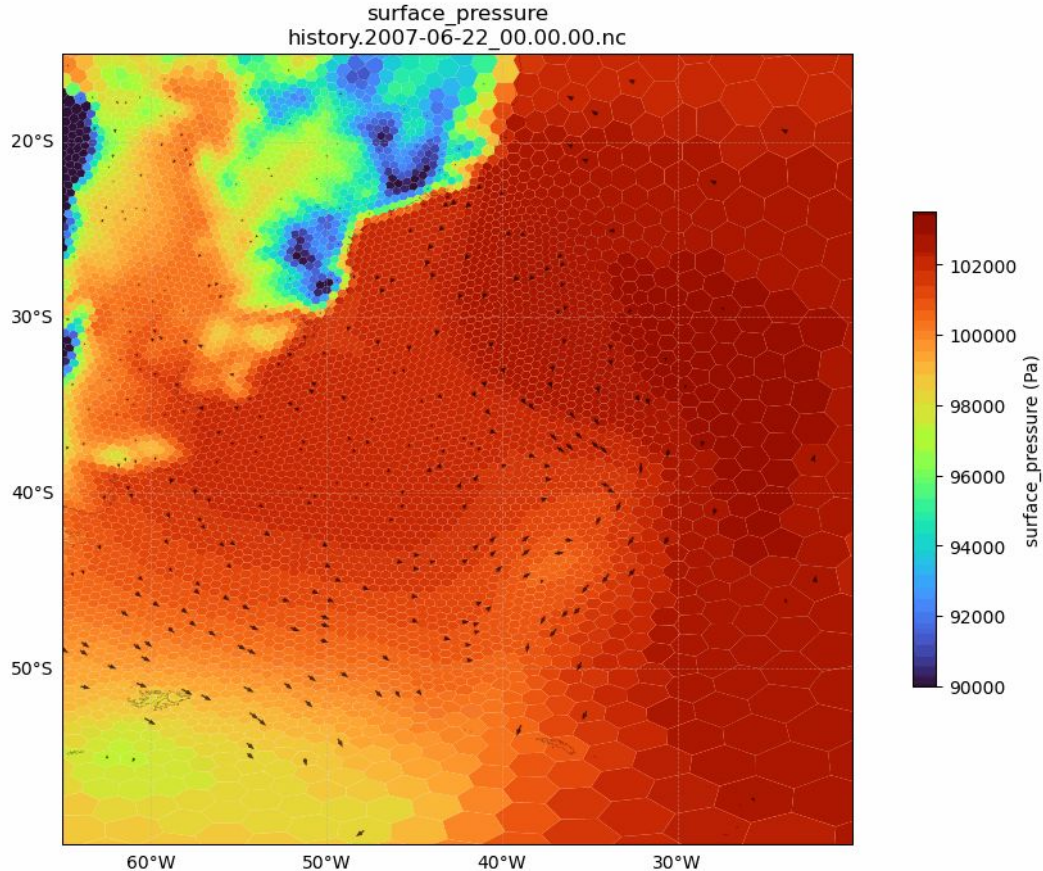
```
# STEP 4: Executing the Model run:  
time ${SCRIPTS}/4.run_model.bash ${EXP} ${MESH} ${YYYYMMDDHH} ${FCST} ${RES}  
#exit
```

```
bash 0.run_all.bash
```

```
&physics  
  config_sst_update = false  
  config_sstdiurn_update = false  
  config_deepsoiltemp_update = false  
  config_radtlw_interval = '00:30:00'  
  config_radtsw_interval = '00:30:00'  
  config_conv_interval = '#CONFIG_CONV_INTERVAL#'  
  config_bucket_update = 'none'  
  config_physics_suite = 'convection_permitting_monan'  
  config_mynn_edmf = 0
```

Physics
parametrizations!

Tutorial 3: Results



That is all for now...

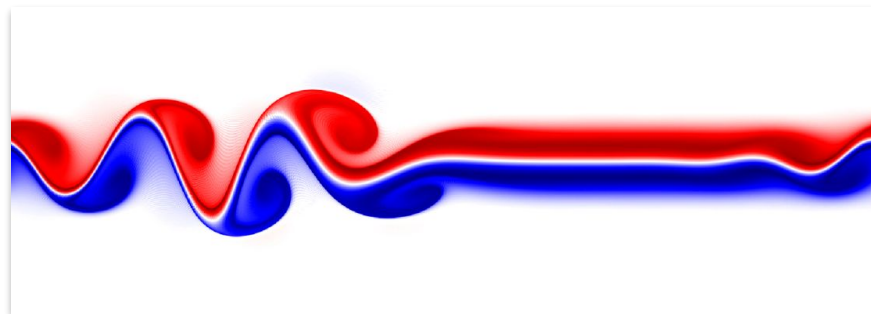
“All models are wrong, but some are useful”

— George Box

More details at: <https://github.com/CGFD-USP/MPAS-References>

And: www.ime.usp.br/~pedrosp

Thanks!



Extra slides

Initial setup / configuration

First, let's create a 200km uniform mesh using Jigsaw.

In the MPAS source code directory:

- Activate the maps-tools conda environment: `conda activate mpas-tools`
- Step into de grids/grids directory: `cd grids/grids`
- Create the grid: `python3 ../utilities/jigsaw/spherical_grid.py -g unif -r 200 -o unif200km`

This will create the *unif200km* directory with the following data:

```
tmpyc_kkvna/      unif200km.jig      unif200km_mpas.nc
unif200km_graph.info  unif200km.log      unif200km.msh
unif200km-HFUN.msh  unif200km-MESH.msh  unif200km_triangles.nc
```

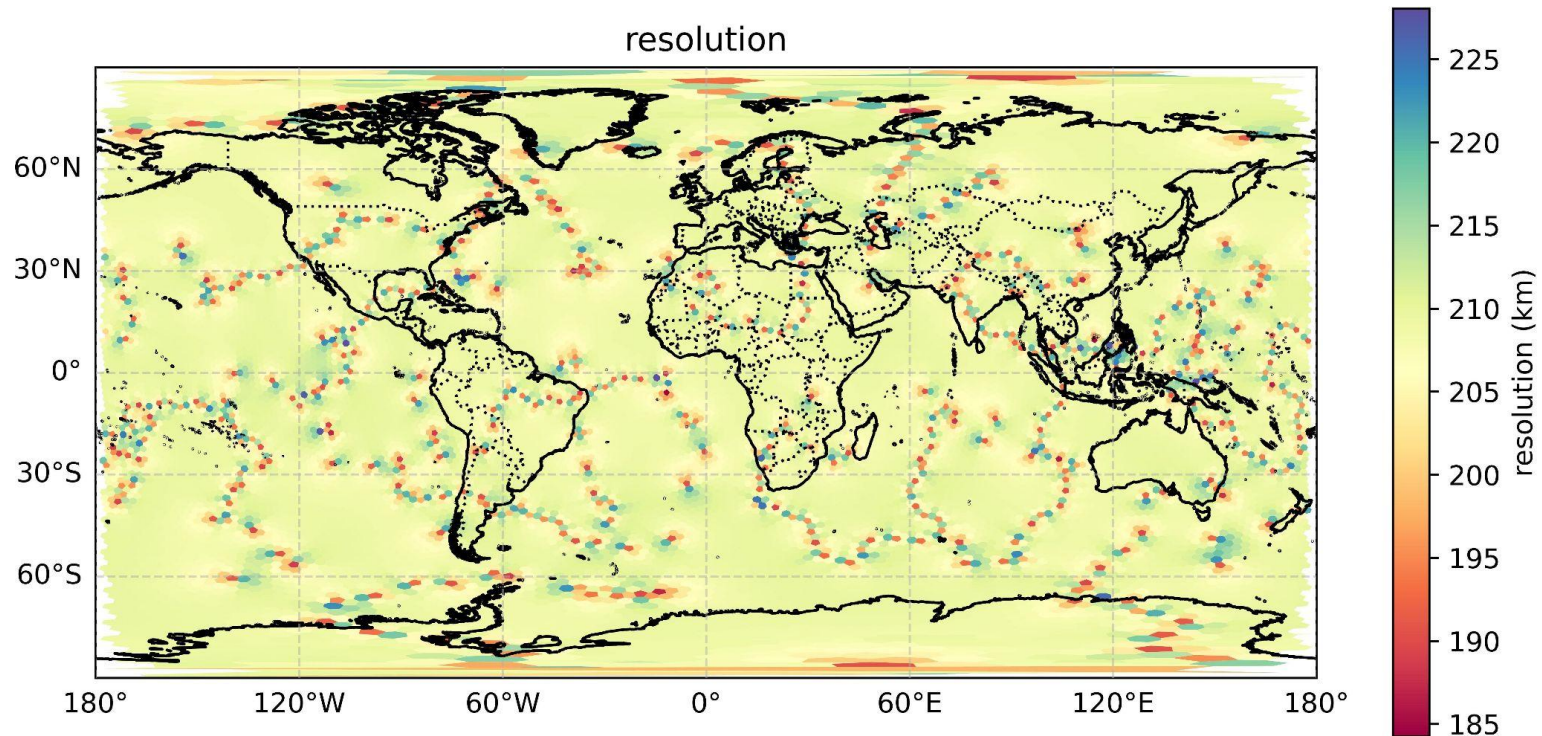
Inside this new directory we partition the grid using *metis* and the *unif200km_graph.info*, for example, for 4 processes: `gpmemis -contig -minconn unif200km_graph.info 4`

This will create the *unif200km_graph.info.part.4* file.

Initial setup / configuration

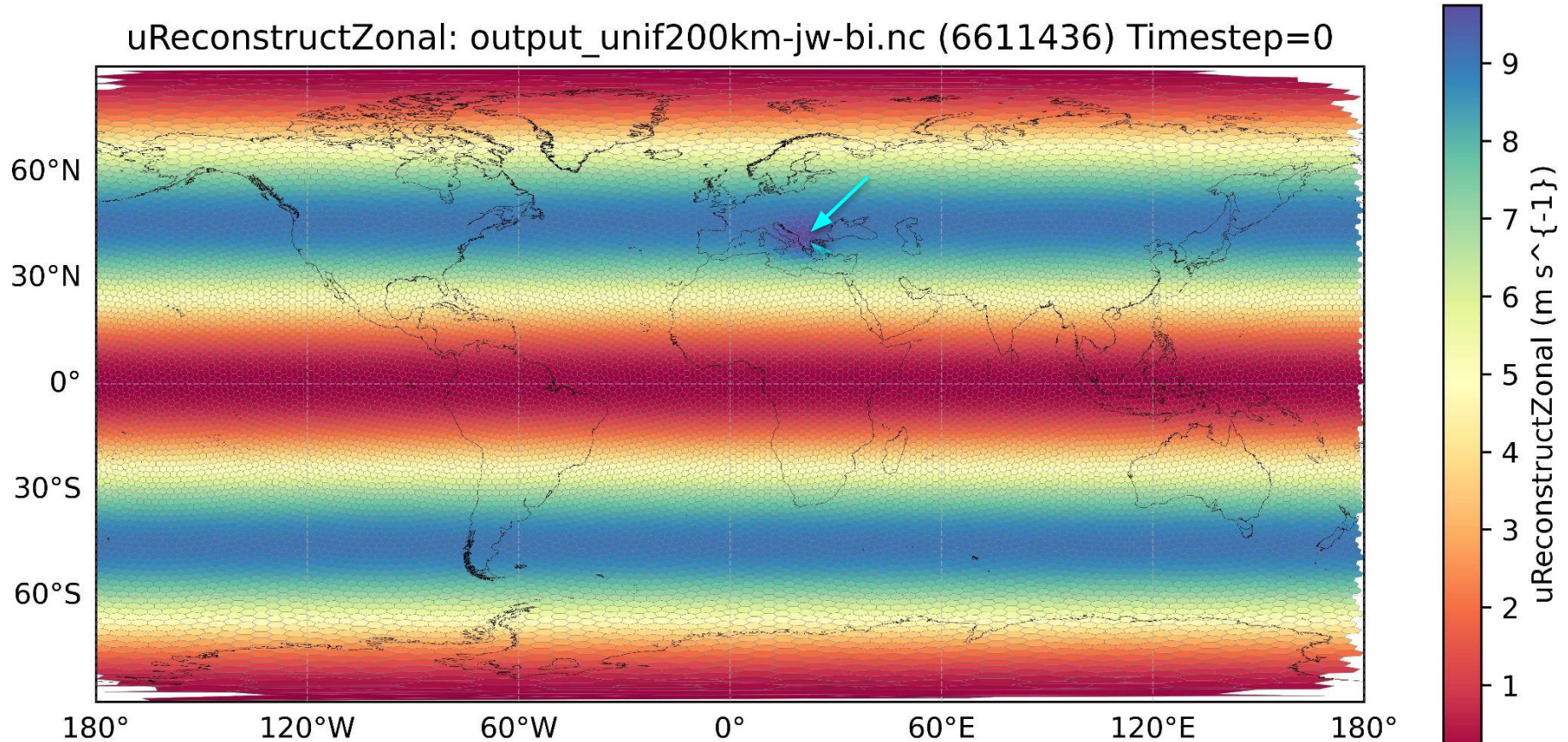
We can check the grid resolution using one of the post-processing scripts from the MPAS-BR repository:

- `python ../../../../post_proc/py/grid_maps/mpas_plot_grid.py -g unif200km_mpas.nc -o unif200km-grid.jpg`



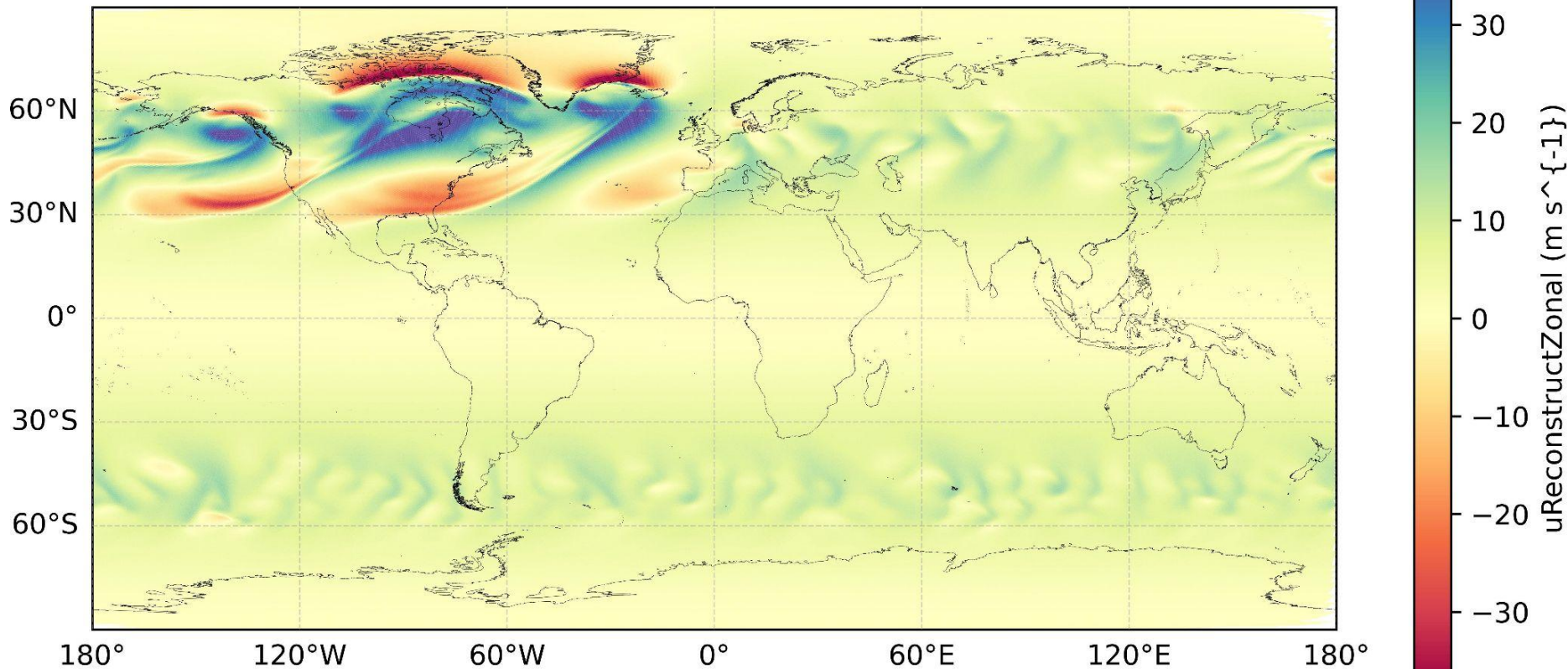
Example Simulations: JW Baroclinic Instability

uReconstructZonal: output_unif200km-jw-bi.nc (6611436) Timestep=0



JW BI case with 50km grid resolution: Zonal wind field at day 13

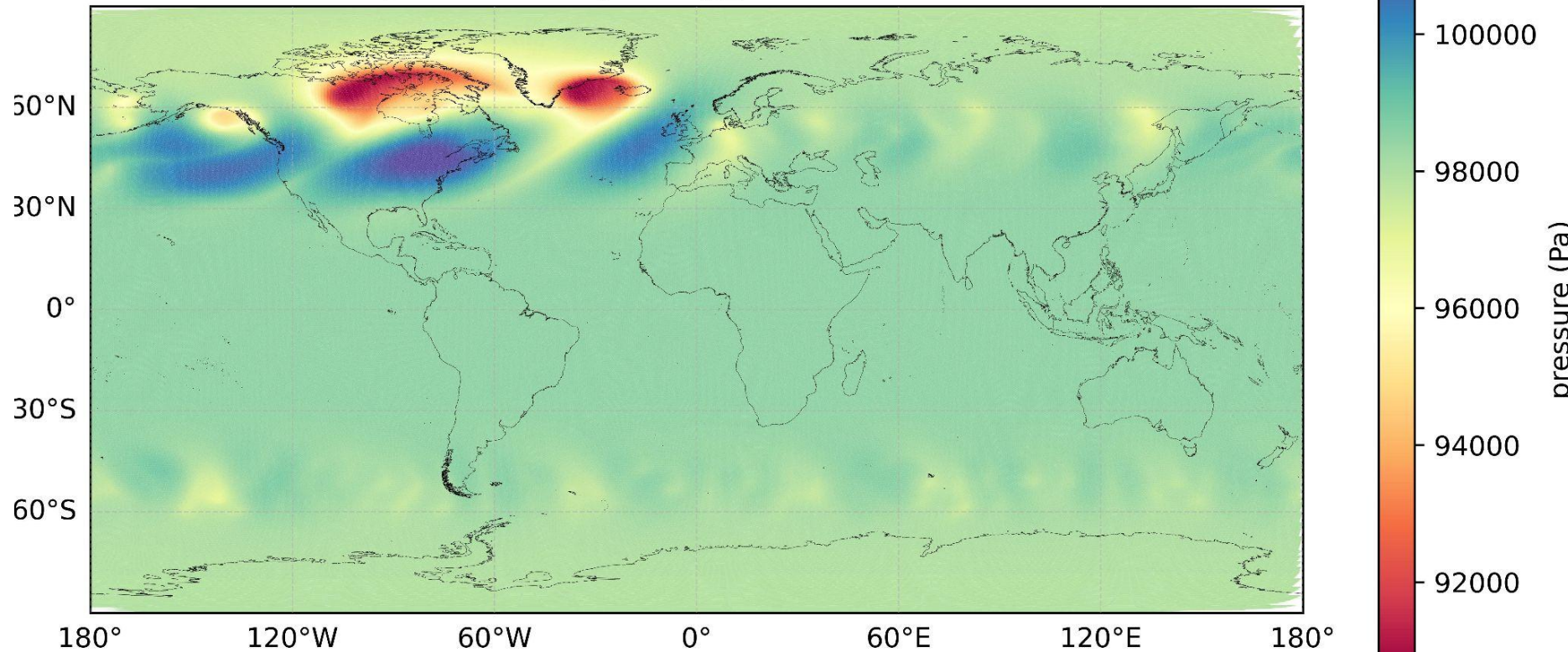
uReconstructZonal: output.day_13.nc (6175494)



Vertical Level 0

JW BI case with 50km grid resolution: Pressure field at day 13

pressure: output.day_13.nc (6175494)



180°

120°W

60°W

0°

60°E

120°E

180°

pressure (Pa)

100000

98000

96000

94000

92000

Vertical Level 0

Example Simulations: JW Baroclinic Instability

In the MPAS root directory create a new runs directory and a directory for this case:

- `mkdir -p runs/jw-bi-200km ; cd runs/jw-bi-200km`

Create a link to the grid files:

- `ln -s ../../grids/grids/unif200km/unif200km_mpas.nc ./`
- `ln -s ../../grids/grids/unif200km/unif200km_graph.info.part.4 ./`

Create a link to the `init_atmosphere` executable:

- `ln -s ../../init_atmosphere ./`

Then, create the *namelist.init_atmosphere* and *streams.init_atmosphere* files

Example Simulations: JW Baroclinic Instability

Run the `init_atmosphere` executable:

- `mpirun -n 4 ./init_atmosphere_model`

Check that the `unif200km_jw-bi.init.nc` file was created. The end of the `log.init_atmosphere.0000.out` should look like:

```
*****
Finished running the init_atmosphere core
*****

Timer information:
  Globals are computed across all threads and processors

Columns:
  total time: Global max of accumulated time spent in timer
  calls: Total number of times this timer was started / stopped.
  min: Global min of time spent in a single start / stop
  max: Global max of time spent in a single start / stop
  avg: Global max of average time spent in a single start / stop
  pct_tot: Percent of the timer at level 1
  pct_par: Percent of the parent timer (one level up)
  par_eff: Parallel efficiency, global average total time / global max total time

timer_name                total      calls      min          max          avg          pct_tot  pct_par  par_eff
1 total time              0.94919      1      0.94870      0.94919      0.94882    100.00    0.00    1.00
2 initialize              0.26813      1      0.26759      0.26813      0.26782    28.25    28.25    1.00
-----
Total log messages printed:
  Output messages =          218
  Warning messages =           7
  Error messages =            0
  Critical error messages =     0
-----
Logging complete.  Closing file at 2023/11/11 15:04:11
```

```

name!list.atmosphere
1 &nhyd_model
2   config_dt = 720.0
3   config_start_time = '0000-01-01_00:00:00'
4   config_run_duration = '16_00:00:00'
5   config_split_dynamics_transport = false
6   config_number_of_sub_steps = 6
7   config_dynamics_split_steps = 1
8   config_h_mom_eddy_visc2 = 0.0
9   config_h_mom_eddy_visc4 = 0.0
10  config_v_mom_eddy_visc2 = 0.0
11  config_h_theta_eddy_visc2 = 0.0
12  config_h_theta_eddy_visc4 = 0.0
13  config_v_theta_eddy_visc2 = 0.0
14  config_horiz_mixing = '2d_smagorinsky'
15  config_len_disp = 200000.
16  config_u_vadv_order = 3
17  config_w_vadv_order = 3
18  config_theta_vadv_order = 3
19  config_scalar_vadv_order = 3
20  config_theta_adv_order = 3
21  config_scalar_adv_order = 3
22  config_scalar_advection = false
23  config_positive_definite = false
24  config_coef_3rd_order = 1.0
25  config_monotonic = false
26  config_epssm = 0.1
27  config_smdiv = 0.1
28 /
29
30 &damping
31   config_zd = 22000.0
32   config_xnutr = 0.0
33 /
34
35 &decomposition
36   config_block_decomp_file_prefix = 'unif200km_graph.info.part.'
37 /
38
39 &restart
40   config_do_restart = false
41 /
42
43 &printout
44   config_print_global_minmax_vel = true
45   config_print_global_minmax_sca = false
46 /
47
48 &physics
49   config_physics_suite = 'none'
50 /

```

```
streams.atmosphere
```

```

1 <streams>
2
3 <immutable_stream name="input"
4   |
5   | type="input"
6   | filename_template="unif200km_jw-bi.init.nc"
7   | input_interval="initial_only"/>
8
9 <immutable_stream name="restart"
10  |
11  | type="input;output"
12  | filename_template="restart.$Y-$M-$D_$h.$m.$s.nc"
13  | input_interval="initial_only"
14  | output_interval="5_00:00:00"/>
15
16 <stream name="output"
17   |
18   | type="output"
19   | filename_template="output_unif200km-jw-bi.nc"
20   | filename_interval="none"
21   | output_interval="24:00:00">
22
23   <file name="stream_list.atmosphere.output"/>
24 </stream>
</streams>

```

```
stream_list.atmosphere.output
```

```

1 latCell
2 lonCell
3 xCell
4 yCell
5 zCell
6 indexToCellID
7 latEdge
8 lonEdge
9 xEdge
10 yEdge
11 zEdge
12 indexToEdgeID
13 latVertex
14 lonVertex
15 xVertex
16 yVertex
17 zVertex
18 indexToVertexID
19 cellsOnEdge
20 nEdgesOnCell
21 nEdgesOnEdge
22 edgesOnCell
23 edgesOnEdge
24 weightsOnEdge
25 dvEdge
26 dcEdge
27 angleEdge
28 areaCell
29 areaTriangle
30 cellsOnCell
31 verticesOnCell
32 verticesOnEdge
33 edgesOnVertex
34 cellsOnVertex
35 kiteAreasOnVertex
36 u
37 w
38 pressure
39 rho
40 theta
41 vorticity
42 ke
43 uReconstructZonal
44 uReconstructMeridional

```

namelist.atmosphere options

The available options for the *namelist.atmosphere* file can be inspected in *src/core_atmosphere/Registry.xml*

```
src > core_atmosphere > Registry.xml > registry > dims
55 <!-- ***** -->
56 <!-- ***** Namelists ***** -->
57 <!-- ***** -->
58
59 <nml_record name="nhyd_model" in_defaults="true">
60 <nml_option name="config_time_integration" type="character" default_value="SRK3" in_defaults="false"
61 units="-"
62 description="Time integration scheme"
63 possible_values="'SRK3'"/>
64
65 <nml_option name="config_time_integration_order" type="integer" default_value="2"
66 units="-"
67 description="Order for RK time integration"
68 possible_values="2 or 3"/>
69
70 <nml_option name="config_dt" type="real" default_value="720.0"
71 units="s"
72 description="Model time step, seconds"
73 possible_values="Positive real values"/>
74
75 <nml_option name="config_calendar_type" type="character" default_value="gregorian" in_defaults="false"
76 units="-"
77 description="Simulation calendar type"
78 possible_values="'gregorian','gregorian_noleap'"/>
79
80 <nml_option name="config_start_time" type="character" default_value="2010-10-23_00:00:00"
81 units="-"
82 description="Starting time for model simulation"
83 possible_values="'YYYY-MM-DD_hh:mm:ss'"/>
84
85 <nml_option name="config_stop_time" type="character" default_value="none" in_defaults="false"
86 units="-"
87 description="Stopping time for model simulation"
88 possible_values="'YYYY-MM-DD_hh:mm:ss'"/>
89
90 <nml_option name="config_run_duration" type="character" default_value="5_00:00:00"
91 units="-"
92 description="Length of model simulation"
93 possible_values="[DDD]_hh:mm:ss"/>
```

Namelist and Streams (links)

Init_atmosphere

- Namelist
- Streams

Atmosphere

- Namelist
- Streams

Run the atmosphere executable:

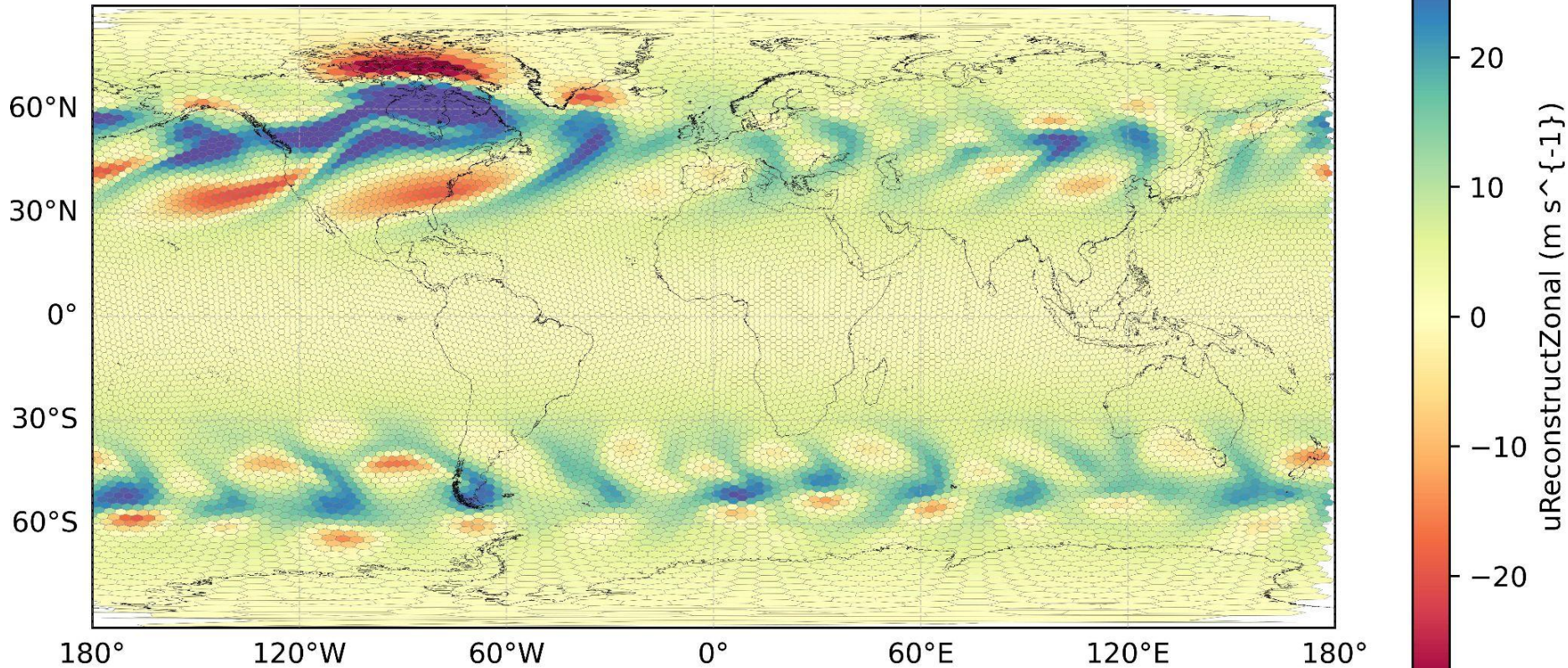
- `mpirun -n 4 ./atmosphere_model`

```
namelist.atmosphere
1  &nhyd_model
2      config_dt = 720.0
3      config_start_time = '0000-01-01_00:00:00'
4      config_run_duration = '16_00:00:00'
5      config_split_dynamics_transport = false
6      config_number_of_sub_steps = 6
7      config_dynamics_split_steps = 1
8      config_h_mom_eddy_visc2 = 0.0
9      config_h_mom_eddy_visc4 = 0.0
10     config_v_mom_eddy_visc2 = 0.0
11     config_h_theta_eddy_visc2 = 0.0
12     config_h_theta_eddy_visc4 = 0.0
13     config_v_theta_eddy_visc2 = 0.0
14     config_horiz_mixing = '2d_smagorinsky'
15     config_len_disp = 20000.
16     config_u_vadv_order = 3
17     config_w_vadv_order = 3
18     config_theta_vadv_order = 3
19     config_scalar_vadv_order = 3
20     config_theta_adv_order = 3
21     config_scalar_adv_order = 3
22     config_scalar_advection = false
23     config_positive_definite = false
24     config_coef_3rd_order = 1.0
25     config_monotonic = false
26     config_epssm = 0.1
27     config_smdiv = 0.1
28 /
29
30 &damping
31     config_zd = 22000.0
32     config_xnutr = 0.0
33 /
34
35 &decomposition
36     config_block_decomp_file_prefix = 'unif200km_graph.info.part.'
37 /
38
39 &restart
40     config_do_restart = false
41 /
42
43 &printout
44     config_print_global_minmax_vel = true
45     config_print_global_minmax_sca = false
46 /
47
48 &physics
49     config_physics_suite = 'none'
50 /
```

```
streams.atmosphere
1  <streams>
2
3      <immutable_stream name="input"
4          type="input"
5          filename_template="unif200km_jw-bi.init.nc"
6          input_interval="initial_only"/>
7
8      <immutable_stream name="restart"
9          type="input;output"
10         filename_template="restart.$Y-$M-$D.$m.$s.nc"
11         input_interval="initial_only"
12         output_interval="5_00:00:00"/>
13
14     <stream name="output"
15         type="output"
16         filename_template="output_unif200km-jw-bi.nc"
17         filename_interval="none"
18         output_interval="24:00:00">
19
20         <file name="stream_list.atmosphere.output"/>
21
22     </stream>
23 </streams>
24 </streams>
```

JW BI case with 200km grid resolution: Zonal wind field at day 13

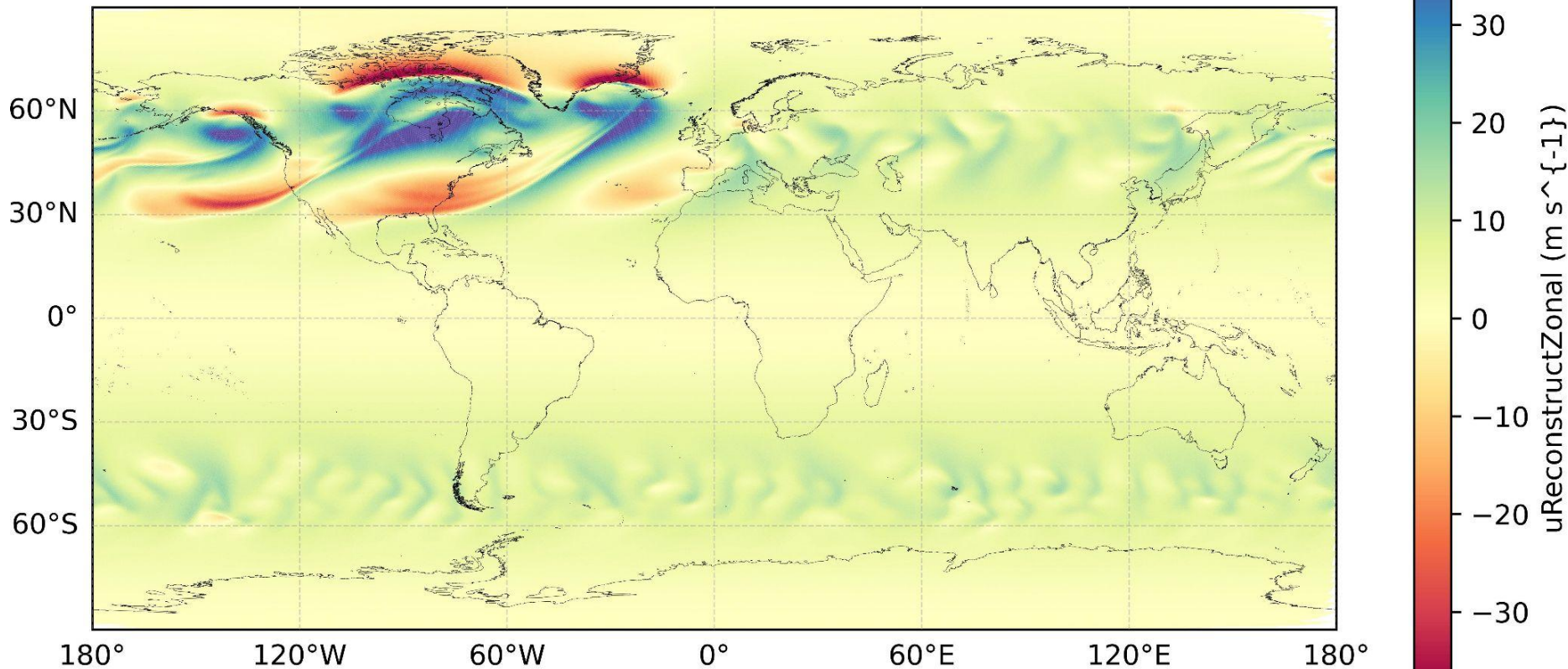
uReconstructZonal: output_unif200km-jw-bi.nc (6611436) Timestep=12



Vertical Level 0

JW BI case with 50km grid resolution: Zonal wind field at day 13

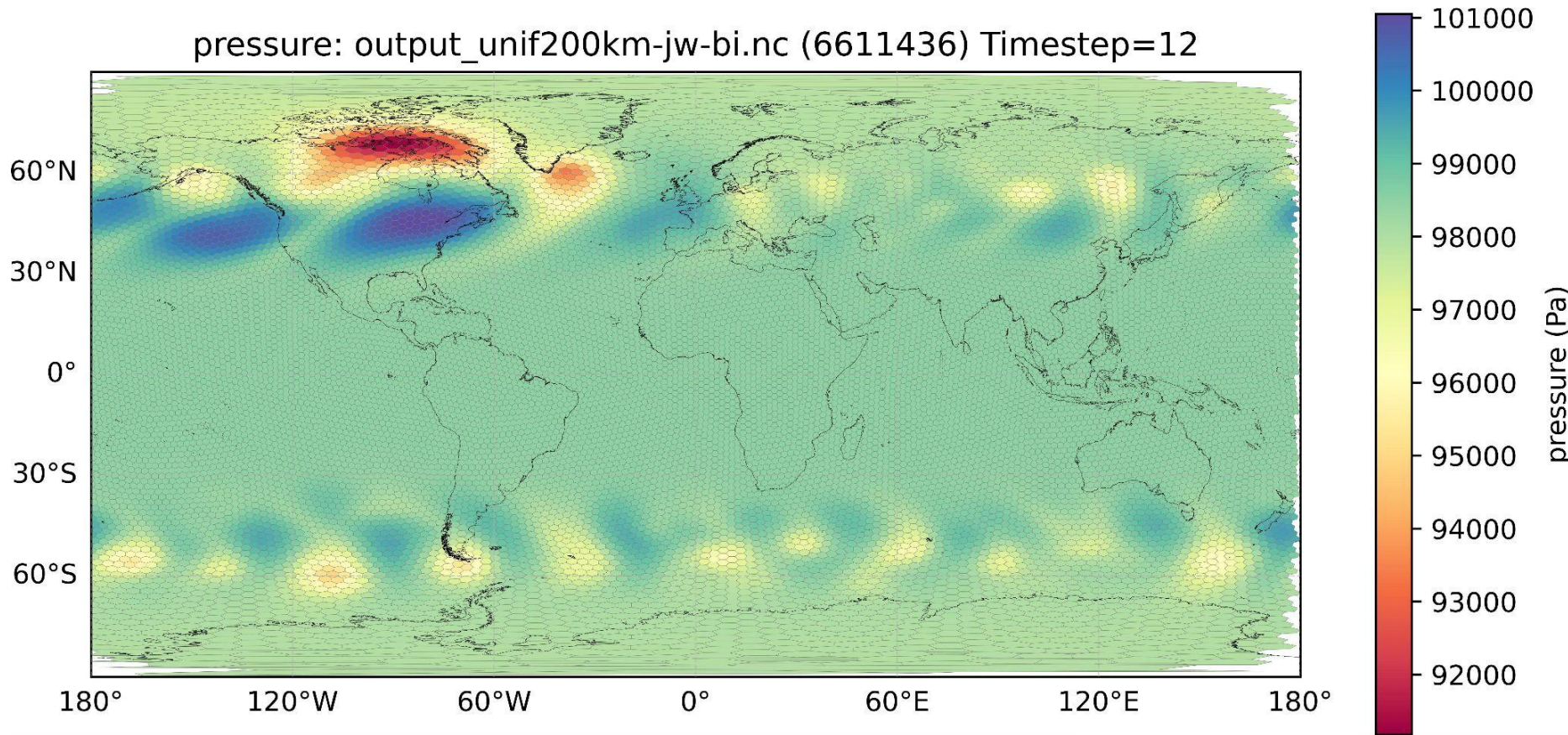
uReconstructZonal: output.day_13.nc (6175494)



Vertical Level 0

JW BI case with 200km grid resolution: Pressure field at day 13

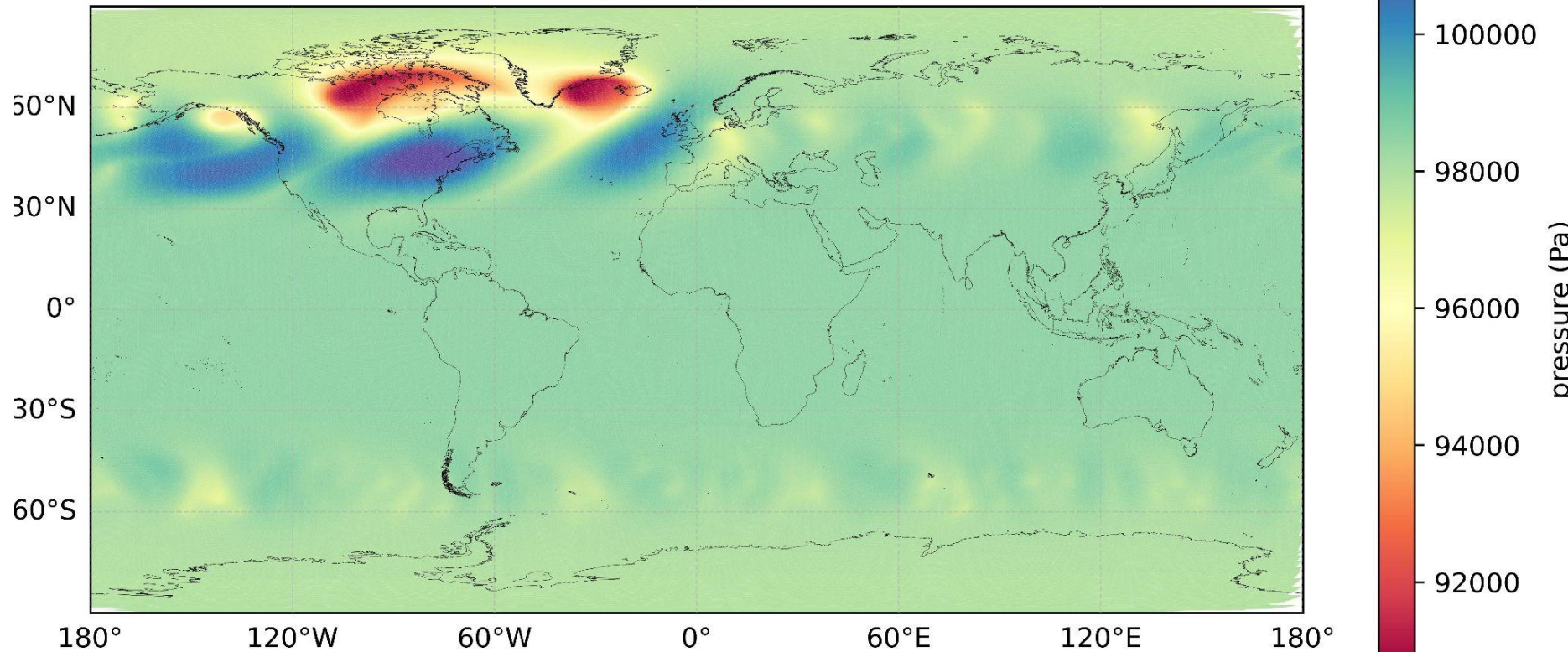
pressure: output_unif200km-jw-bi.nc (6611436) Timestep=12



Vertical Level 0

JW BI case with 50km grid resolution: Pressure field at day 13

pressure: output.day_13.nc (6175494)



180°

120°W

60°W

0°

60°E

120°E

180°

pressure (Pa)

100000

98000

96000

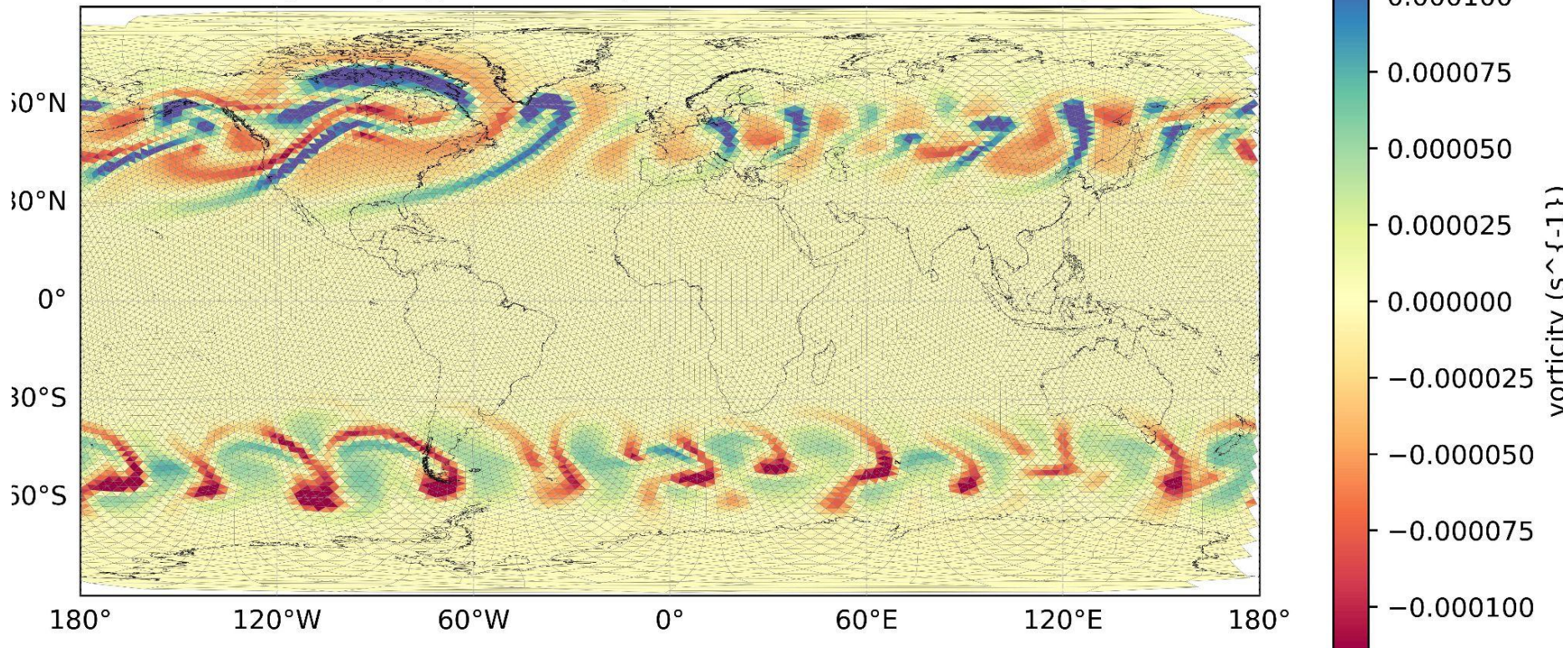
94000

92000

Vertical Level 0

JW BI case with 200km grid resolution: Vorticity field at day 13

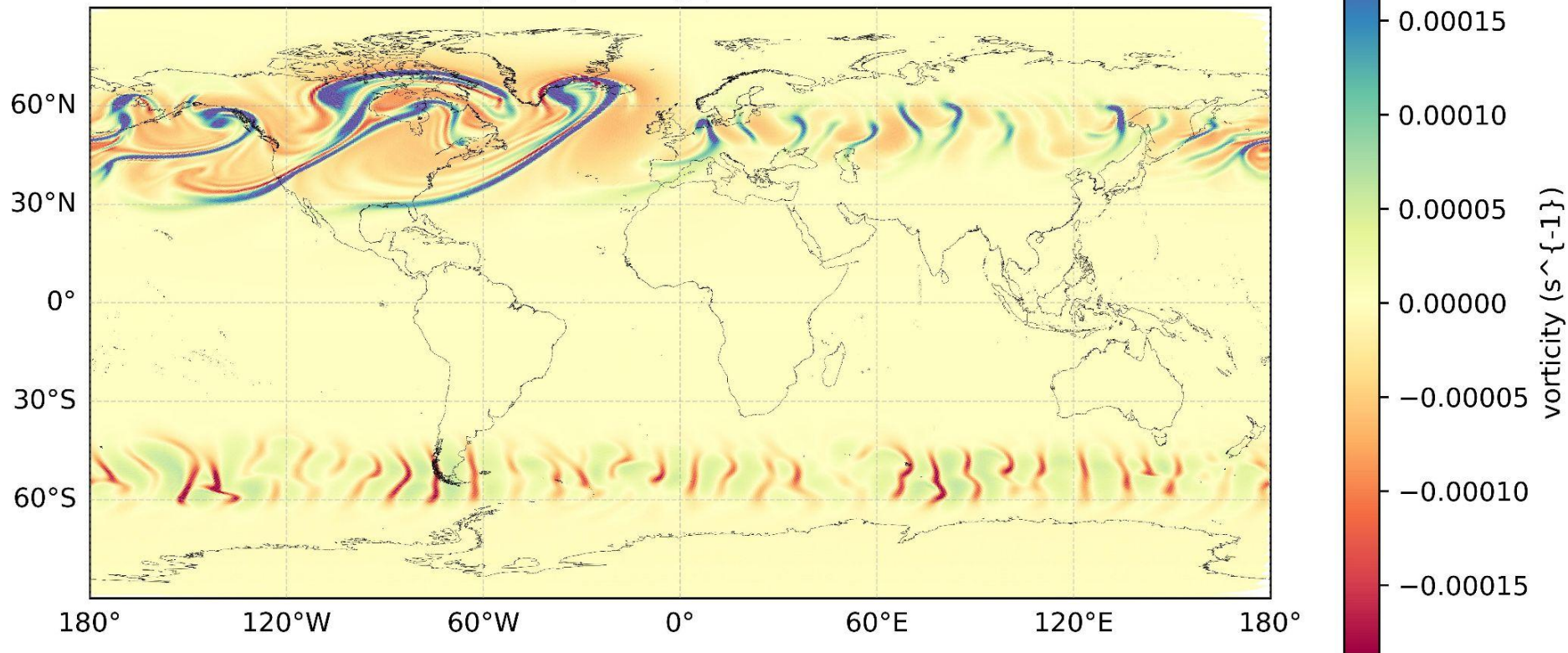
vorticity: output_unif200km-jw-bi.nc (13221104) Timestep=12



Vertical Level 0

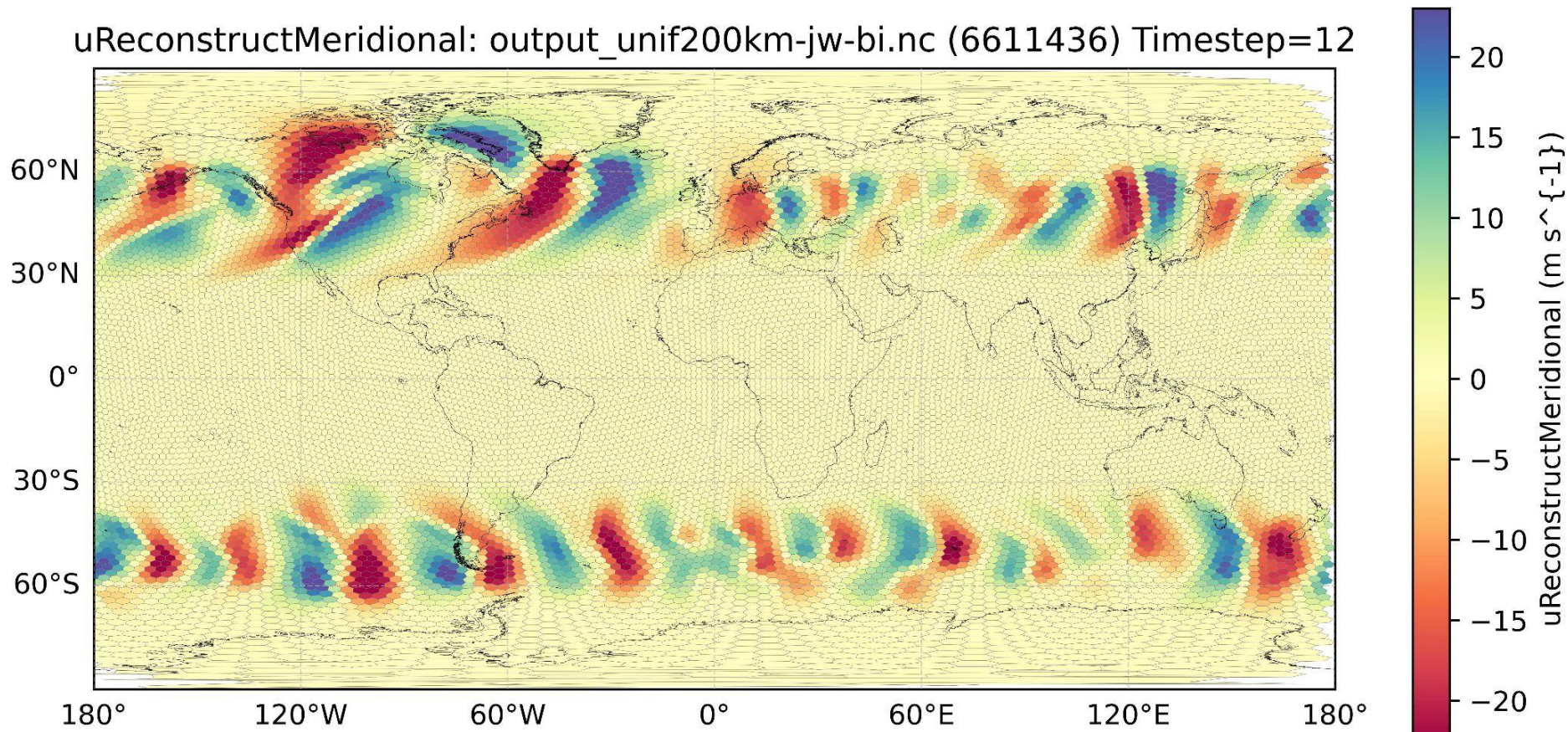
JW BI case with 50km grid resolution: Vorticity field at day 13

vorticity: output.day_13.nc (12350884)



JW BI case with 200km grid resolution: Meridional wind field at day 13

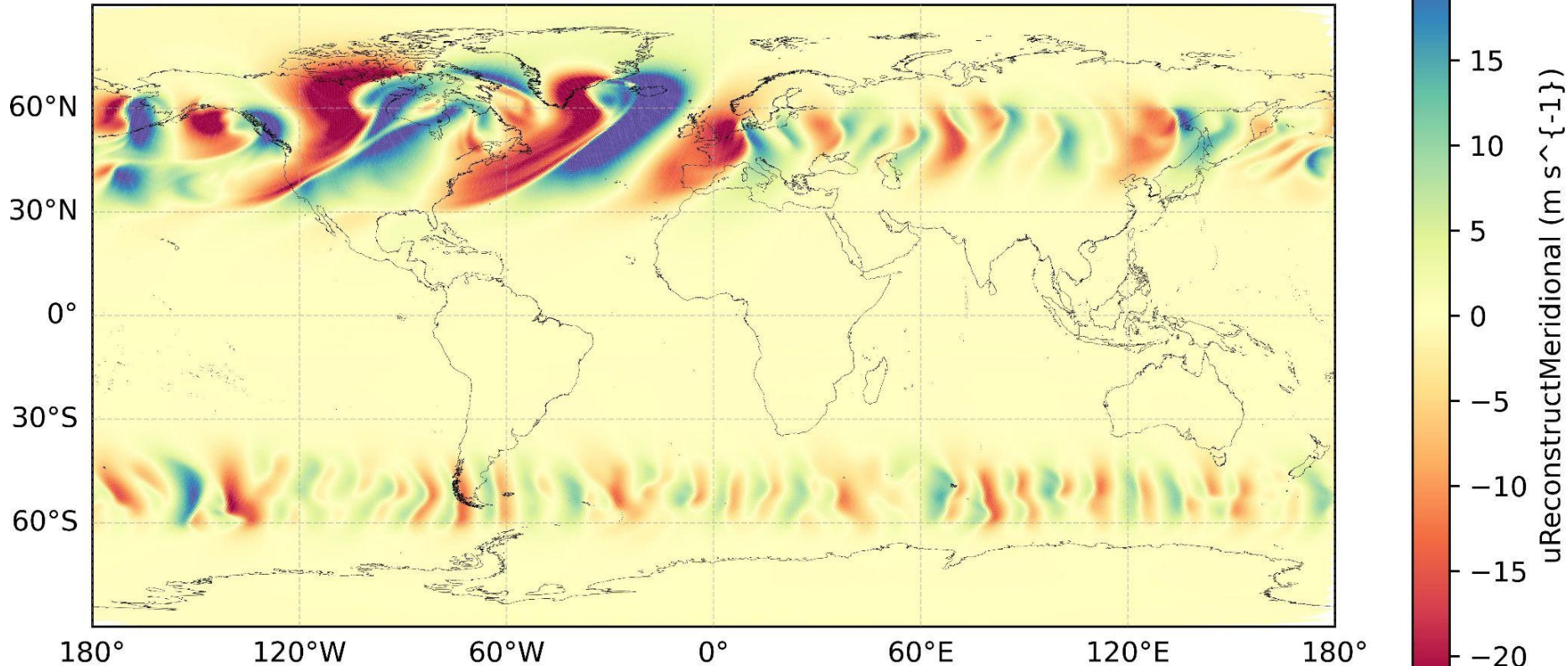
uReconstructMeridional: output_unif200km-jw-bi.nc (6611436) Timestep=12



Vertical Level 0

JW BI case with 50km grid resolution: Meridional wind field at day 13

uReconstructMeridional: output.day_13.nc (6175494)



Vertical Level 0

DCMIP2016

Non-hydrostatic dynamical core design and intercomparison

https://www.earthsystemgrid.org/project/DCMIP_2016.html

Short name	Long name	Modeling center or group
ACME-A	Atmosphere model of the Accelerated Climate Model for Energy	Sandia National Laboratories and University of Colorado, Boulder, USA
CSU	Colorado State University Model	Colorado State University, USA
DYNAMICO	DYNAMical core on the ICOSahedron	Institut Pierre Simon Laplace (IPSL), France
FV ³	GFDL Finite-Volume Cubed-Sphere Dynamical Core	Geophysical Fluid Dynamics Laboratory, USA
FVM	Finite Volume Module of the Integrated Forecasting System	European Centre for Medium-Range Weather Forecasts
GEM	Global Environmental Multiscale model	Environment and Climate Change Canada, Canada
ICON	ICOSahedral Non-hydrostatic model	Max-Planck-Institut für Meteorologie, Germany
MPAS	Model for Prediction Across Scales	National Center for Atmospheric Research, USA
NICAM	Non-hydrostatic Icosahedral Atmospheric Model	AORI/JAMSTEC/AICS, Japan
OLAM	Ocean Land Atmosphere Model	Duke University/University of Miami, USA
Tempest	Tempest Non-hydrostatic Atmospheric Model	University of California, Davis, USA

<https://gmd.copernicus.org/articles/10/4477/2017/>

DCMIP2016

Non-hydrostatic dynamical core design and intercomparison

Short name	Equation set	Prognostic variables	Horizontal grid	Numerical method	Horizontal staggering
ACME-A	H/NH	$\mathbf{u}_h, w, \rho_s, \rho_s \theta, \Phi, \rho_s q_i$	Cubed sphere (Sect. 3.2)	SE	A grid
CSU	NH (unified)	$\zeta, D, w, p_s, \theta_v, q_i$	Geodesic (Sect. 3.4)	FV	Z grid
DYNAMICO	H/NH	$\mathbf{v}_h, \rho_s w, \rho_s, \rho_s \theta_v, \Phi, \rho_s q_i$	Geodesic (Sect. 3.4)	FV	C grid
FV ³	NH	$\mathbf{u}_h, w, \rho_s, \rho_s \theta_v, \Phi, \rho_s q_i$	Cubed sphere (Sect. 3.2)	FV	D grid
FVM	NH (D)	$\rho_d, \mathbf{u}_h, w, \theta', q_i$	Octahedral (Sect. 3.6)	FV	A grid
GEM	NH	$\mathbf{u}_h, w, \zeta, T_v, p, q_i$	Yin–Yang (Sect. 3.7)	FD	C grid
ICON	NH (D)	$\mathbf{u}_h, w, \rho, \theta_v, \rho q_i$	Icosahedral triangular (Sect. 3.3)	FV	C grid
MPAS	NH	$\rho_d \mathbf{u}_h, \rho_d w, \rho_d, \rho_d \theta_v, \rho_d q_i$	CCVT (Sect. 3.5)	FV	C grid
NICAM	NH	$\rho \mathbf{u}_h, \rho w, \rho, \rho e, \rho q_i$	Geodesic (Sect. 3.4)	FV	A grid
OLAM	NH (D)	$\rho \mathbf{u}_h, \rho w, \rho, \rho \theta_{il}, \rho q_i$	Geodesic (Sect. 3.4)	FV	C grid
Tempest	NH	$\mathbf{u}_h, w, \rho, \rho \theta_v, \rho q_i$	Cubed sphere (Sect. 3.2)	SE	A grid

<https://gmd.copernicus.org/articles/10/4477/2017/>

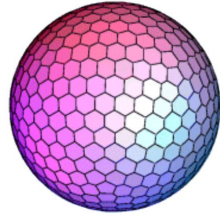
DCMIP2016

Non-hydrostatic dynamical core design and intercomparison

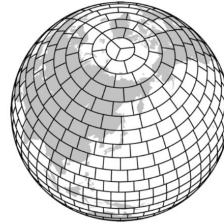
DCMIP-2016 Models



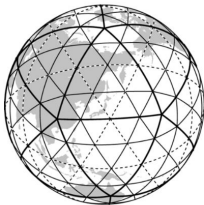
- ACME (E3SM) (DoE, CU)
- FV3 (GFDL)
- Tempest (UC Davis)



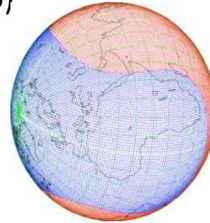
- CSU_LZ (CSU)
- OLAM (U. Miami)
- NICAM (Riken, U. Tokyo)
- MPAS (NCAR)



- FVM (ECMWF)



- ICON (DWD & MPI, Germany)
- DYNAMICO (LMD, IPSL, France), hydrostatic



- GEM (Environment Canada)

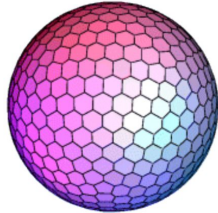
DCMIP2016

Non-hydrostatic dynamical core design and intercomparison

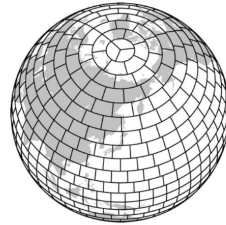
DCMIP-2016 Models



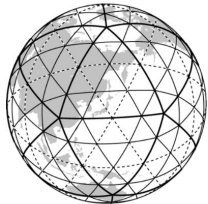
- ACME (E3SM) (DoE, CU)
- FV3 (GFDL)
- Tempest (UC Davis)



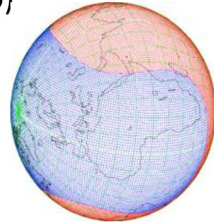
- CSU_LZ (CSU)
- OLAM (U. Miami)
- NICAM (Riken, U. Tokyo)
- MPAS (NCAR)



- FVM (ECMWF)



- ICON (DWD & MPI, Germany)
- DYNAMICO (LMD, IPSL, France), hydrostatic

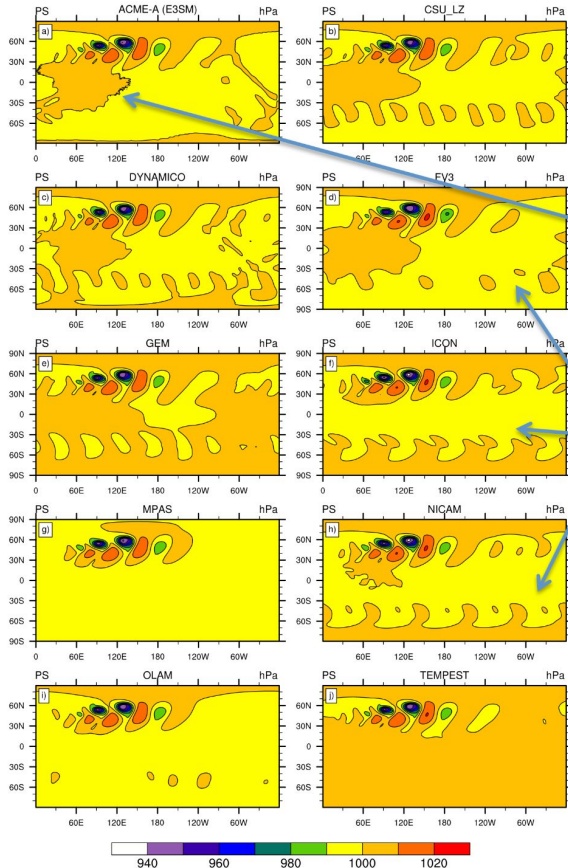


- GEM (Environment Canada)

<https://www2.cesm.ucar.edu/events/wg-meetings/2018/presentations/amwg/jablonowski.pdf>

DCMIP2016

Snapshots of the **dry** baroclinic wave



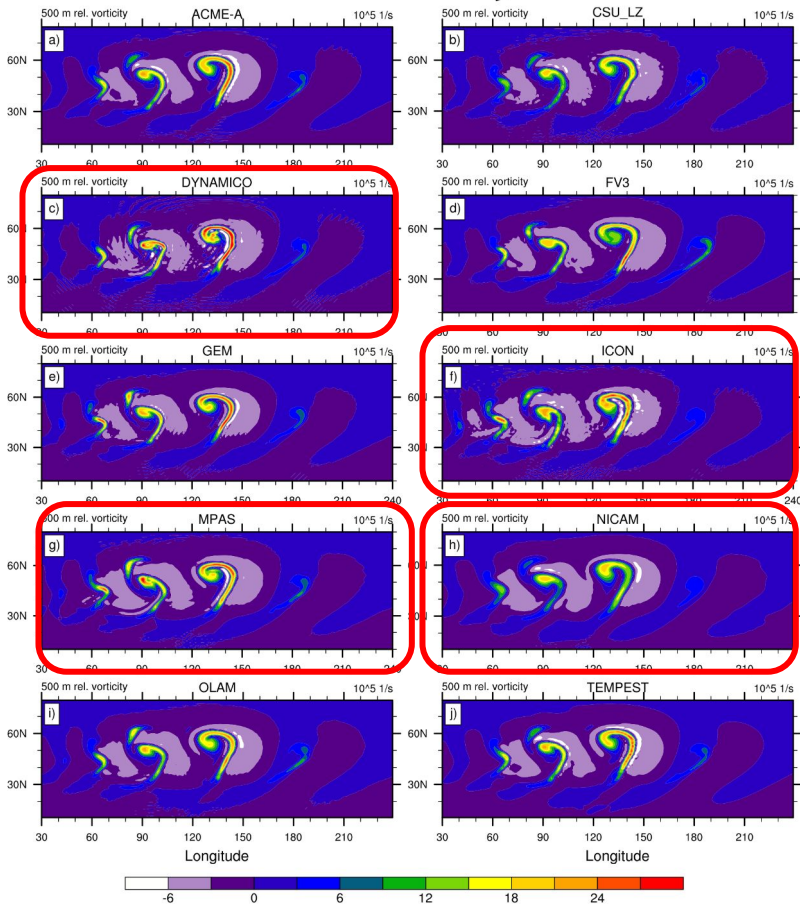
Surface pressure at day 10 ($\Delta x=110$ km): overall patterns similar, details differ

- Some Gibb's ringing in ACME
- Some grid imprinting (wave 4 and wave 5 signals) in CSU_LZ, DYNAMICO, FV3, ICON, NICAM, apparent in the Southern Hemispheres

<https://www2.cesm.ucar.edu/events/wg-meetings/2018/presentations/amwg/jablonowski.pdf>

DCMIP2016

Relative vorticity in the **moist** baroclinic wave



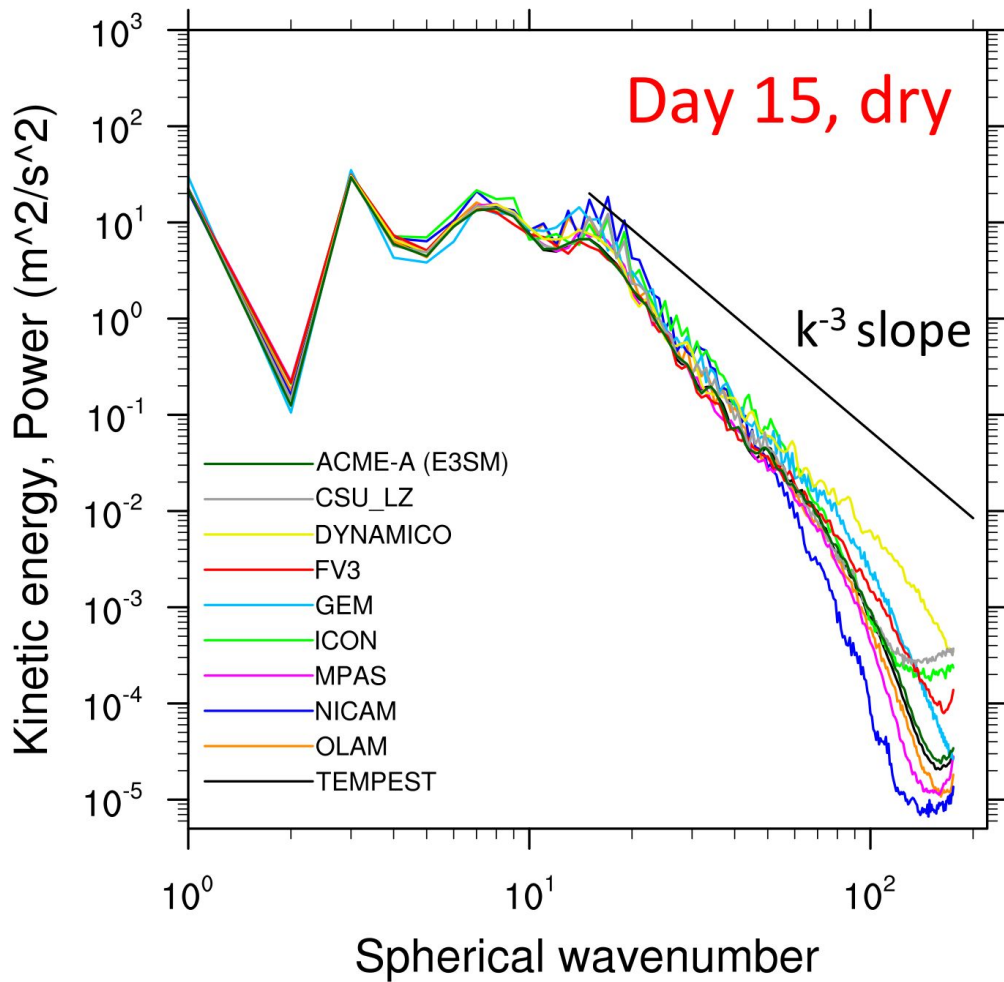
500 m relative vorticity at day 10 ($\Delta x=110$ km): overall patterns similar, details differ

- Maxima and minima differ (by about 30%) and are found in very narrow strips (challenges the 110 km grid spacing)
- Vorticity highlights noise and the diffusive properties of the model

<https://www2.cesm.ucar.edu/events/wg-meetings/2018/presentations/amwg/jablonowski.pdf>

DAY 15 at 1500 m (dry)

DCMIP2016



<https://www2.cesm.ucar.edu/events/wg-meetings/2018/presentations/amwg/jablonowski.pdf>

HIWPP - High Impact Weather Prediction Project

Non-hydrostatic dynamical core tests: Results from idealized test cases

The “new” NCEP Global Model, replacing Spectral (2019)

Participating Dynamical Cores:

The five candidate dycores are listed below, with sponsors in parentheses.

- **FV3** (GFDL) – Cubed sphere grid, finite-volume discretization. (non-hydrostatic version of the hydrostatic core described in Lin, 2004).
- **MPAS** (NCAR) – Unstructured grid with C-grid variable staggering (Skamarock et al, 2012).
- **NEPTUNE** (NRL) – Flexible cubed sphere or icosahedral grid using a spectral element discretization with the Non-hydrostatic Unified Model of the Atmosphere (NUMA) core (Giraldo et al, 2014).
- **NIM** (ESRL) – Non-hydrostatic Icosahedral Model (unstaggered finite-volume A-grid implementation).
- **NMMUJ** (EMC) – Finite-difference cubed-sphere grid version of the B-grid lat/lon grid core described in Janjic and Gall (2012). The construction of the ‘uniform jacobian’ cubed sphere grid is described in NCEP office note 467, available at <http://www.lib.ncep.noaa.gov/ncepofficenotes/2010s/>.

120 km, 60 levels

Baroclinic wave test case at day 9

15 km, 60 levels

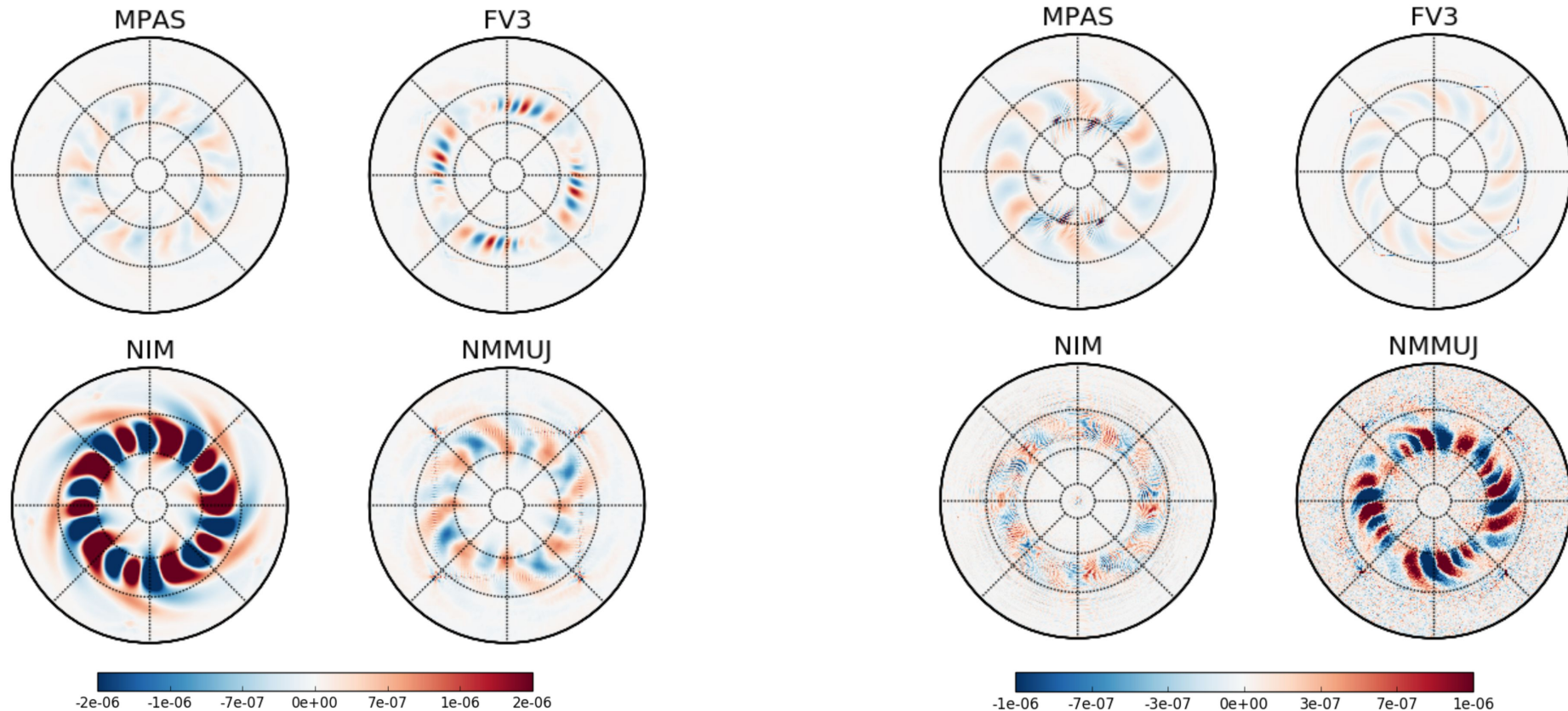
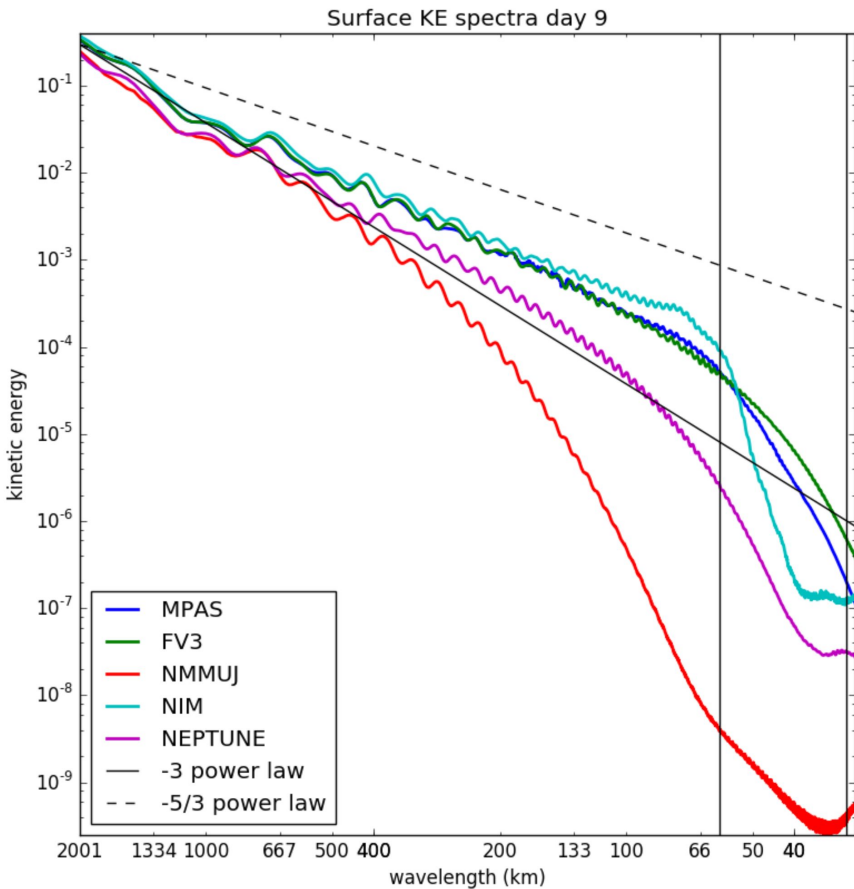


Figure 1: Plots of Southern Hemisphere relative vorticity at 850hPa (with the zonal mean removed) for the high-resolution (nominally 15-km, with 60 levels) baroclinic wave test case at day 9. The outer edge of the plot is 20 degrees south latitude.

Figure 8: Near surface global kinetic energy spectra (m^2/s^2) for day 9 of the 15-km/60 level baroclinic wave test case solutions. The x-axis is wavelength in km, with values ranging from total wavenumber 20 (~ 2000 km) to wavenumber 1440 (~ 28 km), with a log-scale in total wavenumber. Two reference lines are plotted, one with a slope corresponding to a -3 power-law



Installation

- MPAS source code can be downloaded directly at:
 - <https://github.com/pedrospeixoto/MPAS-BR/archive/refs/heads/master.zip>
- Alternatively, the repository can be cloned using git in a terminal (you may wish to FORK to your account):
 - `git clone https://github.com/pedrospeixoto/MPAS-BR.git`
- To compile the code fortran mpi compilers are needed. Working options are described in the file "Makefile". The most straightforward option is to use gfortran and OpenMPI, which can be installed on a Debian system with the terminal command:
 - `sudo apt install gfortran libopenmpi-dev`
- MPAS uses the NetCDF file format for its input and output. Software libraries to work with those files are needed. MPAS accepts a few different libraries for that purpose. We've found the most robust option to be using the PNetCDF library together with new built-in SMIOL code. PNetCDF can be installed on a Debian system with the terminal command:
 - `sudo apt install pnetcdf-bin libpnetcdf-dev`

SMIOL comes bundled with MPAS source code and needs not to be installed.

Compilation

- The MPAS Atmosphere model is divided into two components (also called cores): the ‘init_atmosphere’ core, used just for initialization of a problem, and the ‘atmosphere’ core, which actually runs the model.
- To compile the ‘init_atmosphere’ core, in a terminal inside the source code directory, issue the following command:
 - `make gfortran CORE=init_atmosphere PNETCDF=/usr`
- The PNETCDF environment variable should point to the directory where PNetCDF was installed, which, on Debian systems, is “/usr” by default. If PNetCDF was installed by other means, the env var should be changed accordingly. If compilation works, the following banner should be visible:

```
*****  
MPAS was built with default double-precision reals.  
Debugging is off.  
Parallel version is on.  
Papi libraries are off.  
TAU Hooks are off.  
MPAS was built without OpenMP support.  
MPAS was built without OpenMP-offload GPU support.  
MPAS was built without OpenACC accelerator support.  
Position-dependent code was generated.  
MPAS was built with .F files.  
The native timer interface is being used  
Using the SMIOL library.  
*****
```

Compilation

- Now, to compile the 'atmosphere' core, in a terminal inside the source code directory, issue the following command:
 - `make gfortran CORE=atmosphere AUTOCLEAN=true PNETCDF=/usr`
- If compilation was successful, the same banner as before should be displayed. After compiling both cores, the contents of the directory should be:

```
atmosphere_model*      Makefile
azure-pipelines.yml    met_data/
benchmarks/            MPAS-BR-contributions.md
build_tables*          MPAS-BR-contributions-PXT.md
CAM_ABS_DATA.DBL@      namelist.atmosphere
CAM_AEROPT_DATA.DBL@  namelist.init_atmosphere
default_inputs/       namelist.sw
doc/                   OZONE_DAT.TBL@
docs/                  OZONE_LAT.TBL@
GENPARM.TBL@          OZONE_PLEV.TBL@
grids/                 post_proc/
init_atmosphere_model* README.md
INSTALL                RRTMG_LW_DATA@
LANDUSE.TBL@           RRTMG_LW_DATA.DBL@
LICENSE                RRTMG_SW_DATA@
local_software/        RRTMG_SW_DATA.DBL@
SOILPARM.TBL@
src/
stream_list.atmosphere.diagnostics
stream_list.atmosphere.output
stream_list.atmosphere.surface
stream_list.atmosphere.validate
streams.atmosphere
streams.init_atmosphere
streams.sw
target_domain.Catarina
target_domain.Petropolis
testing_and_setup/
validate/
variables_list
VEGPARM.TBL@
```

Grid Recall

Load environment

- `$conda activate maps-tools`

Create an example grid with Jigsaw

- `(mpas-tools) /.../MPAS-BR/grids/utilities/jigsaw$ python3 spherical_grid.py -g unif -o unif240km -r 240 -l 240`

Check the grid created (resolutions) - Plot.

- `(mpas-tools) /.../MPAS-BR/post_proc/py/grid_maps$ python3 mpas_plot_grid.py -g ../../../../grids/utilities/jigsaw/unif240km/unif240km_mpas.nc -o ../../../../grids/utilities/jigsaw/unif240km/unif240km_mpas.jpg`

Grid partitions

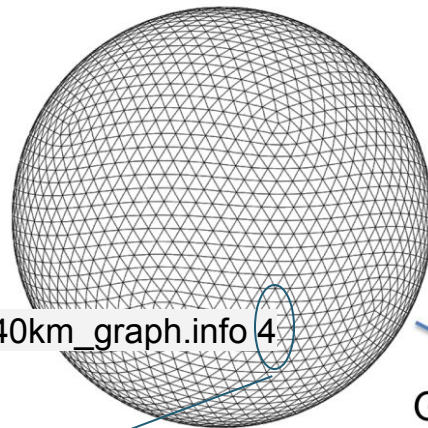
Grid partitions for parallel processing are done using Metis

```
sudo apt-get install metis
```

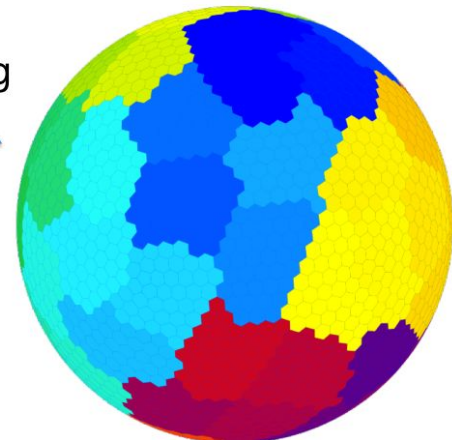
Create partitions. Ex:

```
gpmets -minconn -contig -niter=200 unif240km_graph.info 4
```

N: number of partitions



Graph partitioning



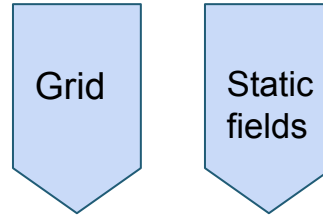
•The *dual* mesh of a Voronoi tessellation is a Delaunay triangulation – essentially the connectivity graph of the cells

•Parallel decomposition of an MPAS mesh then becomes a graph partitioning problem: **equally distribute nodes among partitions (give each process equal work) while minimizing the edge cut (minimizing parallel communication)**

We use the Metis package for parallel graph decomposition

- Currently done as a pre-processing step, but could be done “on-line”
- Fortunately, Metis runs quickly, and a partitioning into n pieces only needs to be done once for a given mesh

Summary



Install/Make

- Clone MPAS repository
- Install gfortran, libopenmpi, libnetcdf
- Make

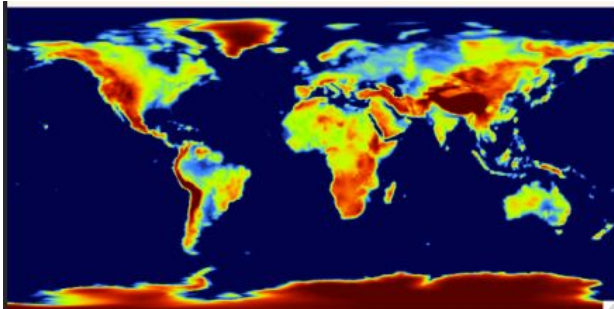
init_atmosphere

- Configure an initial condition (idealized or real data)
- Initialize the atmosphere model
- Set grid and static fields

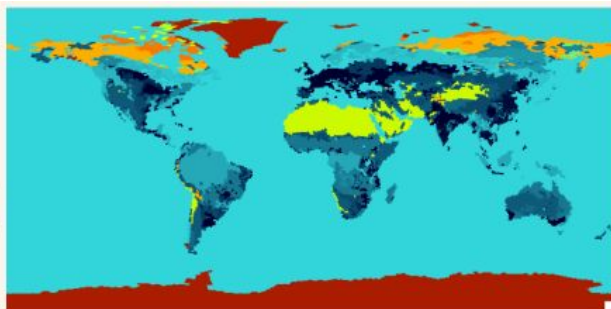
atmosphere

- Run the atmospheric model based on the ini_atmosphere output
- Post process

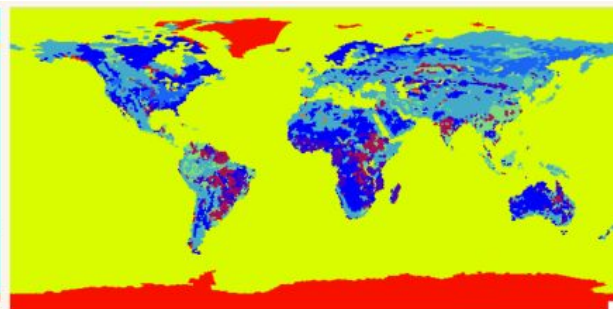
Static fields (examples)



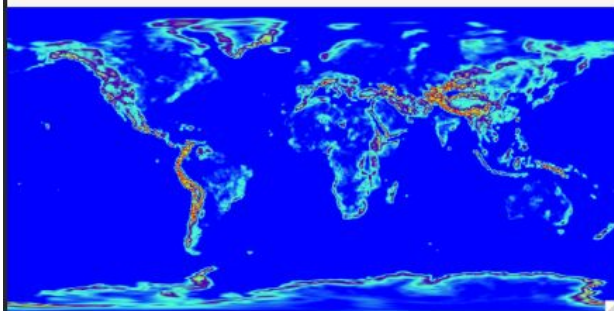
Terrain elevation



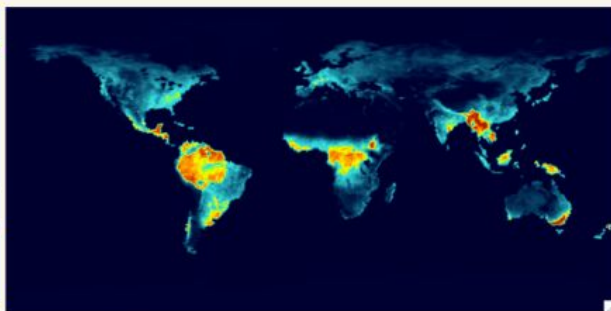
Dominant land cover category



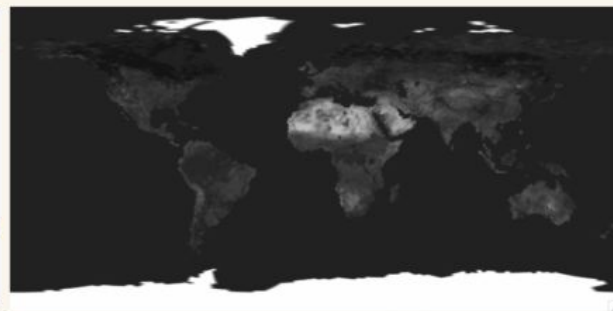
Dominant soil category



Sub-grid-scale terrain variance



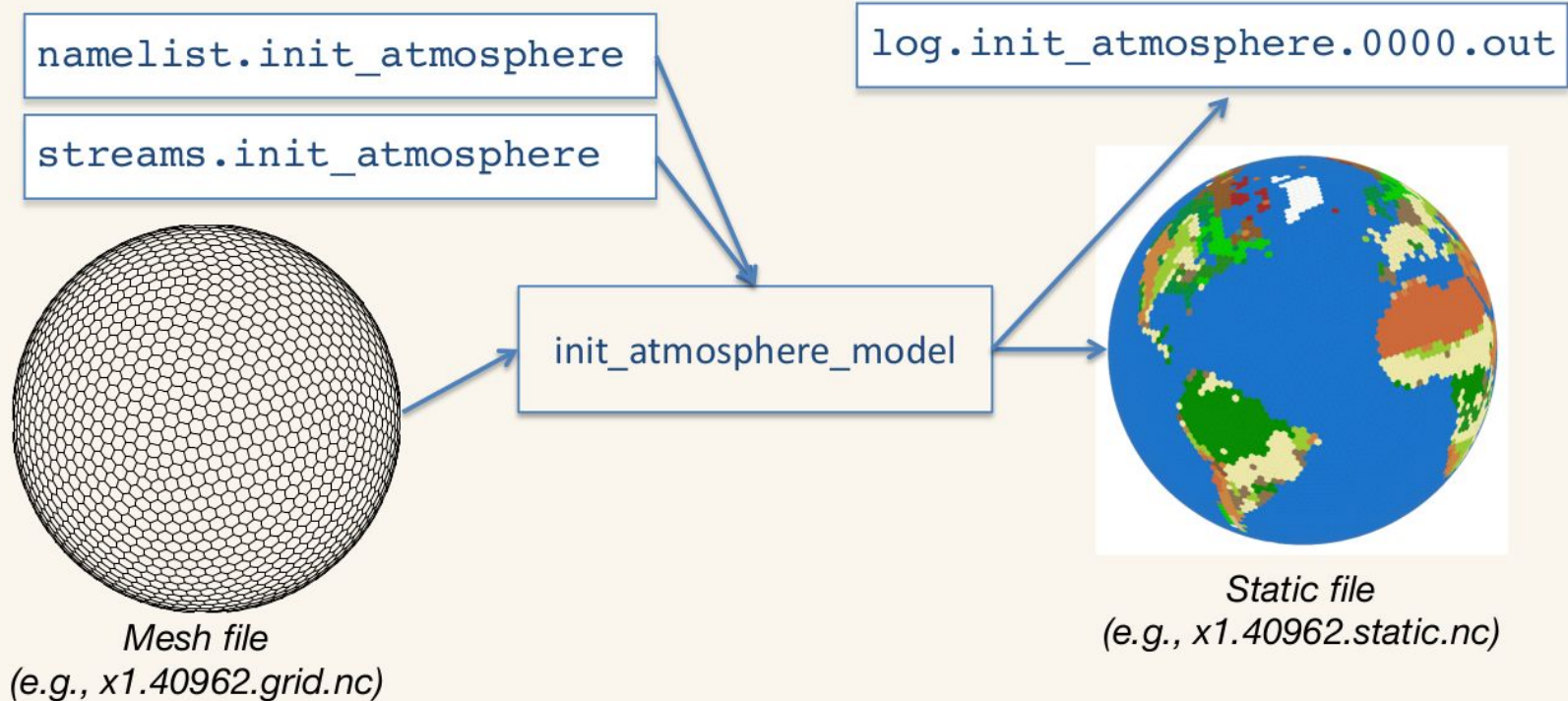
*Climatological monthly
vegetation fraction*



*Climatological monthly surface
albedo*

Pre-processing

Input and output files when producing a “static” file:

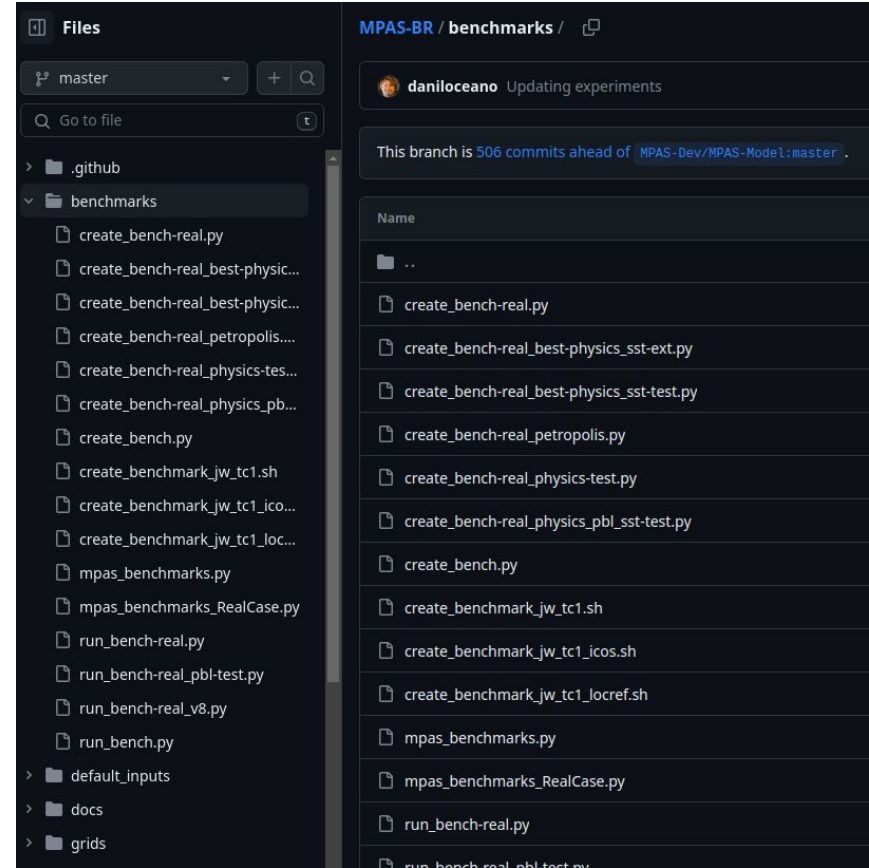


Config init_atmosphere

You can hand tune the namelists/streams and streams of MPAS

or

... we can use Python based scripts to create necessary files to initialize MPAS-A.



Idealized test case

Namelist

```
benchmarks > monan-class-example > jw_baroclinic_wave > ≡ namelist.init_atmosphere
1  &hyd_model
2  |   config_start_time = '0000-01-01_00:00:00'
3  |   config_init_case = 2
4  /
5
6  &dimensions
7  |   config_nvertlevels = 26
8  /
9
10 &decomposition
11 |   config_block_decomp_file_prefix = 'x1.40962.graph.info.part.'
12 /
```

src/core_init_atmosphere/mpas_init_atm_cases.F

```
if ((config_init_case == 1) .or. (config_init_case == 2) .or. (config_init_case == 3)) then
    call mpas_log_write(' Jablonowski and Williamson baroclinic wave test case ')
    if (config_init_case == 1) call mpas_log_write(' no initial perturbation ')
    if (config_init_case == 2) call mpas_log_write(' initial perturbation included ')
    if (config_init_case == 3) call mpas_log_write(' normal-mode perturbation included ')
    block_ptr => domain % blocklist
```

Idealized test case

Streams

(related files)

Input grid file

Output initialization file
For MPAS run

```
benchmarks > monan-class-example > jw_baroclinic_wave > streams.init_atmosphere
1 <streams>
2 <immutable_stream name="input"
3 | | | | type="input"
4 | | | | filename_template="x1.40962.grid.nc"
5 | | | | input_interval="initial_only" />
6
7 <immutable_stream name="output"
8 | | | | type="output"
9 | | | | filename_template="x1.40962.init.nc"
10 | | | | packages="initial_conds"
11 | | | | output_interval="initial_only" />
12
13 <immutable_stream name="surface"
14 | | | | type="output"
15 | | | | filename_template="sfc_not_needed_for_jw"
16 | | | | filename_interval="none"
17 | | | | packages="sfc_update"
18 | | | | output_interval="86400" />
19
20 <immutable_stream name="lbc"
21 | | | | type="output"
22 | | | | filename_template="lbc_not_needed_for_jw"
23 | | | | filename_interval="output_interval"
24 | | | | packages="lbcs"
25 | | | | output_interval="none" />
26
27 </streams>
28
```

Run init_atmosphere

Run the `init_atmosphere` executable:

- `mpirun -n 4 ./init_atmosphere_model`

Must match the number of partitions set with metis!

Netcdf output (mpas input file)

Use either `ncdump -h` or `ncmpidump -h` to quickly inspect the file created:

```
(mpas-tools) pedrosp@ppeixoto:~/ppstorage/Simulations/MPAS-BR/benchmarks/monan-class-example/jw_baroclinic_wave$ ncdump x1.40962.init.nc -h
netcdf x1.40962.init {
dimensions:
    nVertLevels = 26 ;
    nCells = 40962 ;
    Time = UNLIMITED ; // (1 currently)
    StrLen = 64 ;
    nEdges = 122880 ;
    nVertices = 81920 ;
    TWO = 2 ;
    maxEdges = 10 ;
    maxEdges2 = 20 ;
    vertexDegree = 3 ;
    R3 = 3 ;
    nMonths = 12 ;
    FIFTEEN = 15 ;
    nVertLevelsP1 = 27 ;
    nSoilLevels = 4 ;
variables:
    double qv(Time, nCells, nVertLevels) ;
        qv:long_name = "Water vapor mixing ratio" ;
        qv:units = "kg kg^{-1}" ;
    double qc(Time, nCells, nVertLevels) ;
        qc:long_name = "Cloud water mixing ratio" ;
        qc:units = "kg kg^{-1}" ;
    double qr(Time, nCells, nVertLevels) ;
```

Log ini_atmosphere

```
benchmarks > monan-class-example > jw_baroclinic_wave > log.init_atmosphere.0000.out
```

```
1 |-----|
2 Beginning MPAS-init_atmosphere Output Log File for task      0 of      1
3   Opened at 2023/10/30 10:29:56
4 |-----|
```

```
5
6
7 MPAS Init-Atmosphere Version 8.0.1
```

```
8
9
10 Output from 'git describe --dirty': v7.3-1045-g31c4868f
```

```
11
12 Compile-time options:
```

```
13   Build target: gfortran
14   OpenMP support: no
15   OpenACC support: no
16   Default real precision: double
17   Compiler flags: optimize
18   I/O layer: SMIOL
```

```
19
20 Run-time settings:
```

```
21   MPI task count: 1
```

```
22
23 Reading namelist from file namelist.init_atmosphere
```

```
24 *** Encountered an issue while attempting to read namelist file
25     The following values will be used for variables :
```

```
26
27     config_geog_data_path = /glade/work/wrfhelp/wrf/geog/geogdata
28     config_met_prefix = CFSR
29     config_sfc_prefix = SST
30     config_fg_interval = 86400
```

```
timer_name          total      calls      mi
1 total time        6.41626    1          6.4
2 initialize        0.78789    1          0.7
```

```
-----
Total log messages printed:
```

```
Output messages =      233
Warning messages =      11
Error messages =         0
Critical error messages =  0
-----
```

```
Logging complete. Closing file at 2023/10/30 10:30:02
```

Vertical Coordinates

```
Jablonowski and Williamson baroclinic wave test case
initial perturbation included
calling test case setup
point 1 in test case setup
hx computation complete
```

```
k, sh, zw, ah = 1 0.000000000000000 0.000000000000000 0.000000000000000
k, sh, zw, ah = 2 0.754292827454554E-02 1730.76923076923 0.108968669152221E-01
k, sh, zw, ah = 3 0.213346229317396E-01 3461.53846153846 0.429570573912
k, sh, zw, ah = 4 0.391942050280822E-01 5192.30769230769 0.943427718327
k, sh, zw, ah = 5 0.603434261963643E-01 6923.07692307692 0.162163576704
k, sh, zw, ah = 6 0.843325018564451E-01 8653.84615384615 0.242716511050
k, sh, zw, ah = 7 0.110857952634291 10384.6153846154 0.331786982582272
k, sh, zw, ah = 8 0.139696986601515 12115.3846153846 0.424980127306677
k, sh, zw, ah = 9 0.170676983453917 13846.1538461538 0.518049998084810
k, sh, zw, ah = 10 0.203659063412730 15576.9230769231 0.607195305432139
k, sh, zw, ah = 11 0.238528335748478 17307.6923076923 0.689295225282555
k, sh, zw, ah = 12 0.275187691979535 19038.4615384615 0.762066370245209
k, sh, zw, ah = 13 0.313553640224657 20769.2307692308 0.824131424617781
k, sh, zw, ah = 14 0.353553390593274 22500.0000000000 0.875000000000000
k, sh, zw, ah = 15 0.395122746149031 24230.7692307692 0.914971756916988
k, sh, zw, ah = 16 0.438204533834279 25961.5384615385 0.944979639374745
k, sh, zw, ah = 17 0.482747409570915 27692.3076923077 0.966396305281607
k, sh, zw, ah = 18 0.528704930040957 29423.0769230769 0.980828974592044
k, sh, zw, ah = 19 0.576034819156969 31153.8461538462 0.989926834556141
k, sh, zw, ah = 20 0.624698379655273 32884.6153846154 0.995221127789069
k, sh, zw, ah = 21 0.674660014851561 34615.3846153846 0.998011762321982
k, sh, zw, ah = 22 0.725886835374279 36346.1538461538 0.999306656308190
k, sh, zw, ah = 23 0.778348332391215 38076.9230769231 0.999812143270829
k, sh, zw, ah = 24 0.832016103534503 39807.6923076923 0.999965697603075
k, sh, zw, ah = 25 0.886863621074329 41538.4615384615 0.999996932988832
k, sh, zw, ah = 26 0.942866034318192 43269.2307692308 0.999999951549997
k, sh, zw, ah = 27 1.000000000000000 45000.0000000000 1.000000000000000
```

src/core_init_atmosphere/mpas_init_atm_cases.F

```
! Metrics for hybrid coordinate and vertical stretching

str = 1.5
zt = 45000.
dz = zt/float(nz1)

call mpas_log_write(' hx computation complete ')

do k=1,nz

  sh(k) is the stretching specified for height surfaces

  sh(k) = (real(k-1)*dz/zt)**str

  to specify specific heights zc(k) for coordinate surfaces,
  input zc(k) and define sh(k) = zc(k)/zt
  zw(k) is the height of zeta surfaces
  zw(k) = (k-1)*dz yields constant dzeta
  and nonconstant dzeta/dz
  zw(k) = sh(k)*zt yields nonconstant dzeta
  and nearly constant dzeta/dz

  zw(k) = float(k-1)*dz
  zw(k) = sh(k)*zt

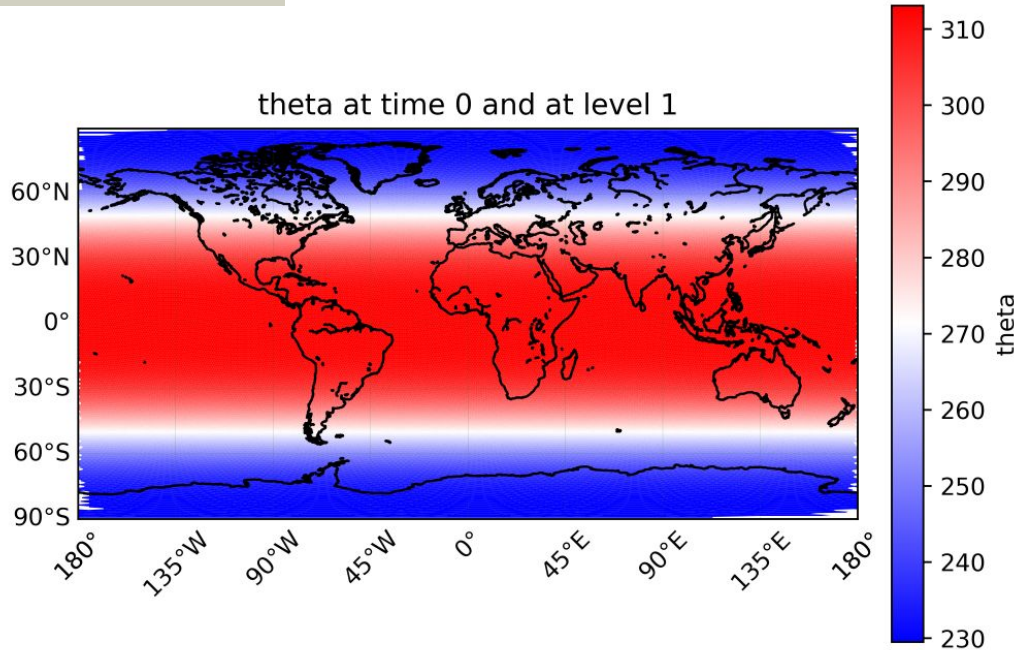
  ah(k) governs the transition between terrain-following
  and pureheight coordinates
  ah(k) = 0 is a terrain-following coordinate
  ah(k) = 1 is a height coordinate

  ah(k) = 1.-cos(.5*pii*(k-1)*dz/zt)**6
  ah(k) = 0.
  call mpas_log_write(' k, sh, zw, ah = $i $r $r $r', intArgs=(/k/), realArgs=(/sh(k),zw(k),ah(k)/))

end do
```

Plot init fields

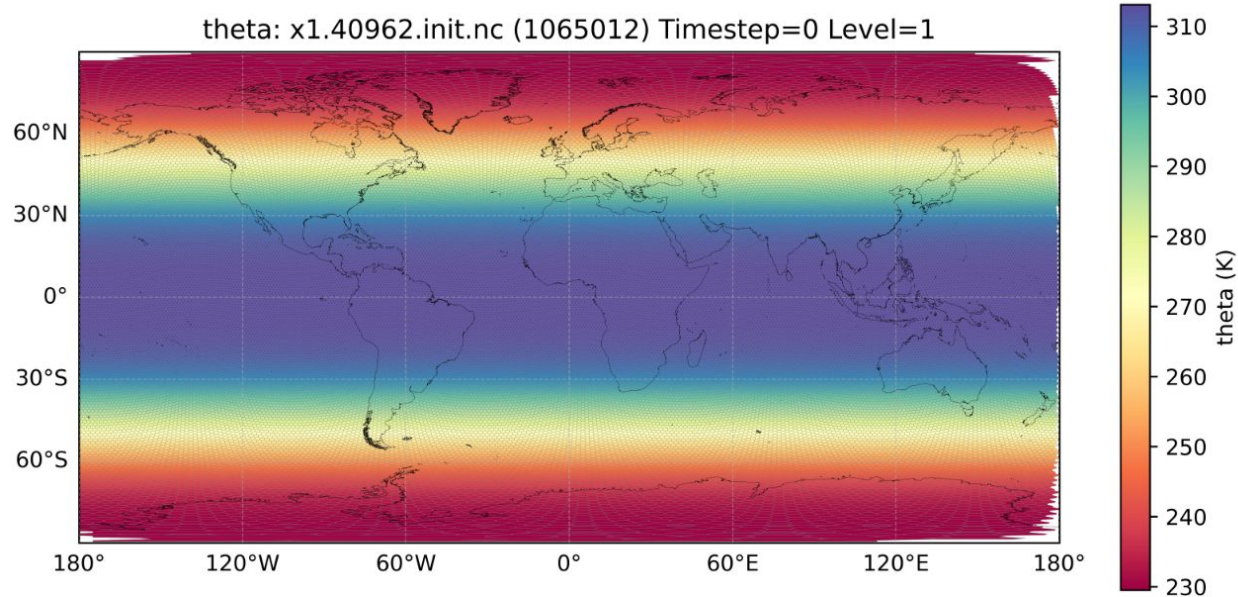
- `MPAS-BR/benchmarks/monan-class-example/jw_baroclinic_wave$ python ../../../../post_proc/py/scalar_lat_lon_2d_plot/mpas_plot_scalar.py -v theta x1.40962.init.nc`



Uses “basemap”
Python library

Plot init fields

- `MPAS-BR/benchmarks/monan-class-example/jw_baroclinic_wave$ python3 ../../../../post_proc/py/plot_scalar_on_native_grid/mpas_plot.py -f x1.40962.init.nc -o x1.40962.init.theta.jpg -v theta -l 0 -t 0`



Uses “cartopy”
Python library

Using Jigsaw grids

With the grid created, we need to create a partition for parallel runs.

- We have a file xx.graph.info from JIGSAW
- Install METIS software for grid partitioning (<http://glaros.dtc.umn.edu/gkhome/views/metis>)
 - `sudo apt-get install metis`
- Run metis partitioning:
 - `gpmmetis -minconn -contig -niter=200 xx.graph.info N`
 - N = number of partitions (nodes/cores to be used)
- Adjust namelist.init_atmosphere and streams.init_atmosphere to point to jigsaw grid.
- Run init_atmosphere_model

```
unif240km_graph.info x
grids > utilities > jigsaw > unif240km > unif240km_graph.info
1 10394 31176
2 10311 10380 9692 10062 9504 10265
3 9405 10097 8173 10014 10015 10255 9759
4 8752 8751 7659 8482 8483 8484 8485
5 9790 9476 9608 9609 9771 9791
6 10394 10341 9291 9825 9570 9571
7 9605 9606 10369 9187 9780 9781 9024
8 10170 10169 10168 9678 10390 10389
9 9802 9051 9834 9835 9836 9837
10 9978 10348 8661 10268 9766 9979
11 10272 9737 9622 8551 10150 10387
12 10344 9664 9663 9662 10097
13 10059 10060 10104 9312 10090 10089
14 9746 9747 9857 8518 9743 9744
15 9123 9122 9121 9847 9848 9832
16 9670 9061 9948 10374 9672 9671
17 9487 9102 10315 10314 10031 9489 9488
18 10172 9575 9574 8514 10165 10114 9371
19 9437 9880 9505 8664 9435 9436
20 9012 10388 9593 9173 10297 10260
21 10083 9862 10373 9564 10120 10340
22 10232 9639 10183 10258 10073 8880
23 9842 9471 10384 9588 9841 9840
24 9806 10312 9695 10316 10343
25 5736 8148 4995 7635 9502 7294 4355
26 8873 9558 10191 10085 10342 10015 |
27 10061 10182 10004 10003 9372
28 9782 8882 10317 9804 9134 9784 9783
29 8670 10385 9876 10382 8144 10006
```

```

(mpas-tools) pedrosp@ppeixoto:~/ppstorage/Simulations/MPAS-BR/grids/utilities/jigsaw/unif240km$ gmetis -minconn -contig -niter=200 unif240km_graph.info
*****
METIS 5.0 Copyright 1998-13, Regents of the University of Minnesota
(HEAD: , Built on: Mar 24 2022, 16:16:52)
size of idx_t: 32bits, real_t: 32bits, idx_t *: 64bits

Graph Information -----
Name: unif240km_graph.info, #Vertices: 10394, #Edges: 31176, #Parts: 4

Options -----
ptype=kway, objtype=cut, ctype=shem, rtype=greedy, iptype=metisrb
dbglvl=0, ufactor=1.030, no2hop=NO, minconn=YES, contig=YES, nooutput=NO
seed=-1, niter=200, ncuts=1

Direct k-way Partitioning -----
- Edgecut: 696, communication volume: 702.

- Balance:
  constraint #0: 1.003 out of 0.000

- Most overweight partition:
  pid: 0, actual: 2606, desired: 2598, ratio: 1.00.

- Subdomain connectivity: max: 3, min: 3, avg: 3.00

- Each partition is contiguous.

Timing Information -----
I/O:                0.002 sec
Partitioning:       0.005 sec (METIS time)
Reporting:          0.001 sec

Memory Information -----
Max memory used:    1.086 MB
*****
(mpas-tools) pedrosp@ppeixoto:~/ppstorage/Simulations/MPAS-BR/grids/utilities/jigsaw/unif240
tmpqtmnn28t      unif240km_graph.info.part.4  unif240km.jig  unif240km-MESH.msh  unif240
unif240km_graph.info  unif240km-HFUN.msh      unif240km.log  unif240km mpas.jpg  unif240

```

```

grids > utilities > jigsaw > unif240km > ≡ unif240km_graph.info.part.4

```

1	1
2	1
3	2
4	2
5	3
6	3
7	3
8	3
9	1
10	0
11	1
12	0
13	3
14	0
15	1
16	1
17	1
18	1
19	1
20	0
21	0
22	2
23	1
24	3
25	1

Convert MPAS data to lat-lon

It is possible to interpolate the MPAS output data to latitude-longitude data, so you can work on your preferred visualization tool....

.... we will not discuss this topic in this class.

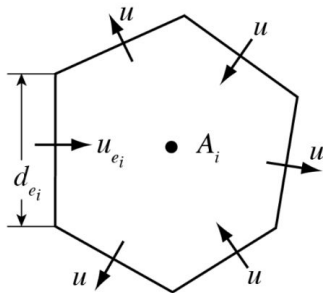
Flux Divergence and Transport

Transport equation, conservative form:
$$\frac{\partial(\rho\psi)}{\partial t} = -\nabla \cdot \mathbf{V}(\rho\psi)$$

Finite-Volume formulation,
Integrate over cell:
$$\int_D \left[\frac{\partial}{\partial t}(\rho\psi) = -\nabla \cdot \mathbf{V}(\rho\psi) \right] dV$$

Apply divergence theorem:
$$\frac{\partial(\overline{\rho\psi})}{\partial t} = -\frac{1}{V} \int_{\Sigma} (\rho\psi) \mathbf{V} \cdot \mathbf{n} d\sigma$$

Discretize in time and space:
$$(\rho\psi)_i^{t+\Delta t} = (\rho\psi)_i^t - \Delta t \frac{1}{A_i} \sum_{n_{e_i}} d_{e_i} \overline{(\rho\mathbf{V} \cdot \mathbf{n}_{e_i})\psi}$$



Velocity divergence operator is 2nd-order accurate for edge-centered velocities.

Flux Divergence and Transport

MPAS uses a Runge-Kutta time-integration scheme.

$$\frac{\partial(\rho\psi)}{\partial t} = L(\mathbf{V}, \rho, \psi)$$

$$(\rho\psi)^* = (\rho\psi)^t + \frac{\Delta t}{3} L(\mathbf{V}, \rho, \psi^t)$$

$$(\rho\psi)^{**} = (\rho\psi)^t + \frac{\Delta t}{2} L(\mathbf{V}, \rho, \psi^*)$$

$$(\rho\psi)^{t+\Delta t} = (\rho\psi)^t + \Delta t L(\mathbf{V}, \rho, \psi^{**})$$

$$(\rho\psi)_i^{t+\Delta t} = (\rho\psi)_i^t - \Delta t \frac{1}{A_i} \sum_{n_{e_i}} d_{e_i} (\rho \mathbf{V} \cdot \mathbf{n}_{e_i}) \psi$$

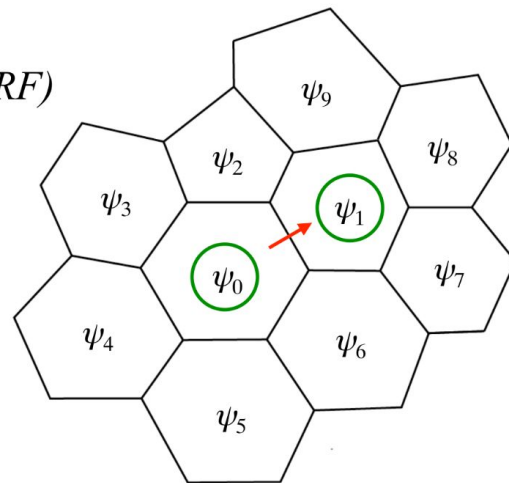
Instantaneous
flux divergence in
RK-based scheme

Computing the flux - consider 1D transport (e.g. from WRF)

$$\frac{\partial(u\psi)_i}{\partial x} = \frac{1}{\Delta x} [F_{i+1/2}(u\psi) - F_{i-1/2}(u\psi)] + O(\Delta x^p).$$

2nd-order

flux: $F(u, \psi)_{i+1/2} = u_{i+1/2} \left[\frac{1}{2} (\psi_{i+1} + \psi_i) \right]$



Flux Divergence and Transport

3rd and 4th-order fluxes:

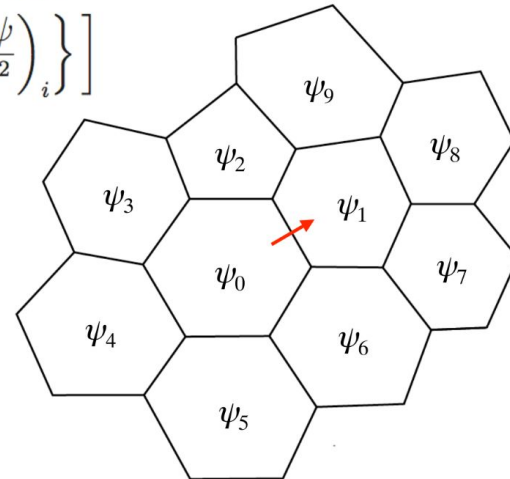
$$F(u, \psi)_{i+1/2} = u_{i+1/2} \left[\frac{1}{2} (\psi_{i+1} + \psi_i) - \frac{1}{12} (\delta_x^2 \psi_{i+1} + \delta_x^2 \psi_i) + \text{sign}(u) \frac{\beta}{12} (\delta_x^2 \psi_{i+1} - \delta_x^2 \psi_i) \right]$$

where $\delta_x^2 \psi_i = \psi_{i-1} - 2\psi_i + \psi_{i+1}$ (Hundsdoerfer et al, 1995; Van Leer, 1985)

Recognizing $\delta_x^2 \psi = \Delta x^2 \frac{\partial^2 \psi}{\partial x^2} + O(\Delta x^4)$ we recast the 3rd and 4th order flux as

$$F(u, \psi)_{i+1/2} = u_{i+1/2} \left[\frac{1}{2} (\psi_{i+1} + \psi_i) - \Delta x_e^2 \frac{1}{12} \left\{ \left(\frac{\partial^2 \psi}{\partial x^2} \right)_{i+1} + \left(\frac{\partial^2 \psi}{\partial x^2} \right)_i \right\} + \text{sign}(u) \Delta x_e^2 \frac{\beta}{12} \left\{ \left(\frac{\partial^2 \psi}{\partial x^2} \right)_{i+1} - \left(\frac{\partial^2 \psi}{\partial x^2} \right)_i \right\} \right]$$

where x is the direction normal to the cell edge and i and $i+1$ are cell centers. We use the least-squares-fit polynomial to compute the second derivatives.



Flux Divergence and Transport

3rd and 4th-order fluxes:

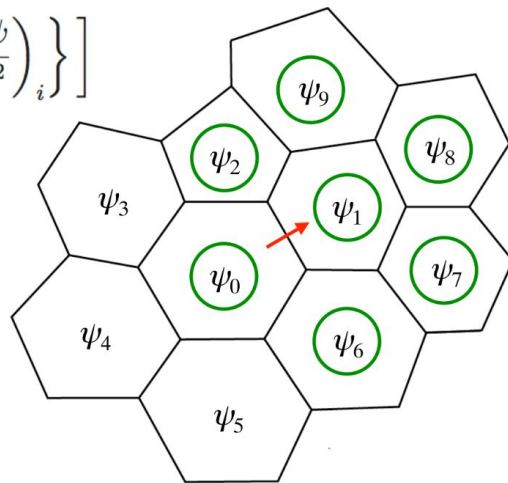
$$F(u, \psi)_{i+1/2} = u_{i+1/2} \left[\frac{1}{2} (\psi_{i+1} + \psi_i) - \frac{1}{12} (\delta_x^2 \psi_{i+1} + \delta_x^2 \psi_i) + \text{sign}(u) \frac{\beta}{12} (\delta_x^2 \psi_{i+1} - \delta_x^2 \psi_i) \right]$$

where $\delta_x^2 \psi_i = \psi_{i-1} - 2\psi_i + \psi_{i+1}$ (Hundsdoerfer et al, 1995; Van Leer, 1985)

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where x is the direction normal to the cell edge and i and $i+1$ are cell centers. We use the least-squares-fit polynomial to compute the second derivatives.



Flux Divergence and Transport

3rd and 4th-order fluxes:

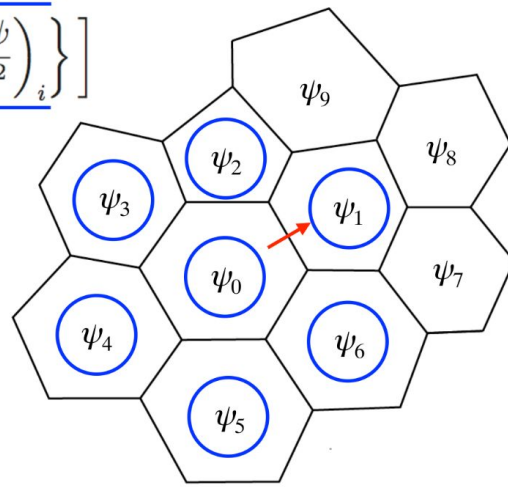
$$F(u, \psi)_{i+1/2} = u_{i+1/2} \left[\frac{1}{2} (\psi_{i+1} + \psi_i) - \frac{1}{12} (\delta_x^2 \psi_{i+1} + \delta_x^2 \psi_i) + \text{sign}(u) \frac{\beta}{12} (\delta_x^2 \psi_{i+1} - \delta_x^2 \psi_i) \right]$$

where $\delta_x^2 \psi_i = \psi_{i-1} - 2\psi_i + \psi_{i+1}$ (Hundsdoerfer et al, 1995; Van Leer, 1985)

Recognizing $\delta_x^2 \psi = \Delta x^2 \frac{\partial^2 \psi}{\partial x^2} + O(\Delta x^4)$ we recast the 3rd and 4th order flux as

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where x is the direction normal to the cell edge and i and $i+1$ are cell centers. We use the least-squares-fit polynomial to compute the second derivatives.



Código (init_atmosphere)

MPAS-BR / src / core_init_atmosphere / mpas_atm_advection.F

Code Blame 941 lines (740 loc) · 31.5 KB Code 55% faster with GitHub Copilot

```
210
211     if (polynomial_order == 2) then
212         na = 6
213         ma = ma+1
214
215         amatrix(1,1) = 1.
216         wmatrix(1,1) = 1.
217         do i=2,ma
218             amatrix(i,1) = 1.
219             amatrix(i,2) = xp(i-1)
220             amatrix(i,3) = yp(i-1)
221             amatrix(i,4) = xp(i-1)**2
222             amatrix(i,5) = xp(i-1) * yp(i-1)
223             amatrix(i,6) = yp(i-1)**2
224
225             wmatrix(i,i) = 1.
226         end do
227
228     else if (polynomial_order == 3) then
229         na = 10
230         ma = ma+1
231
232         amatrix(1,1) = 1.
233         wmatrix(1,1) = 1.
234         do i=2,ma
235             amatrix(i,1) = 1.
236             amatrix(i,2) = xp(i-1)
237             amatrix(i,3) = yp(i-1)
238
239             amatrix(i,4) = xp(i-1)**2
240             amatrix(i,5) = xp(i-1) * yp(i-1)
241             amatrix(i,6) = yp(i-1)**2
242
243             amatrix(i,7) = xp(i-1)**3
244             amatrix(i,8) = yp(i-1) * (xp(i-1)**2)
245             amatrix(i,9) = xp(i-1) * (yp(i-1)**2)
```

Polynomial fit

Pre-calculations for high-order flux

```
368 ! do j=1,n
369 !
370 !     deriv_two(j,1,iEdge) = 2.*xe(iEdge)*xe(iEdge)*bmatrix(4,j) &
371 !                         + 2.*xe(iEdge)*ye(iEdge)*bmatrix(5,j) &
372 !                         + 2.*ye(iEdge)*ye(iEdge)*bmatrix(6,j)
373 !
374 !     end do
375 !
376 !     if (iCell == cellsOnEdge(1,iEdge)) then
377 !         do j=1,n
378 !             deriv_two(j,1,iEdge) = 2.*cos2t*bmatrix(4,j) &
379 !                                 + 2.*costsint*bmatrix(5,j) &
380 !                                 + 2.*sin2t*bmatrix(6,j)
381 !         end do
382 !     else
383 !         do j=1,n
384 !             deriv_two(j,2,iEdge) = 2.*cos2t*bmatrix(4,j) &
385 !                                 + 2.*costsint*bmatrix(5,j) &
386 !                                 + 2.*sin2t*bmatrix(6,j)
387 !         end do
388 !     end if
```

Código (atmosphere)

Flux calculation

MPAS-BR / src / core_atmosphere / dynamics / mpas_atm_time_integration.F

Code Blame 6607 lines (5328 loc) · 293 KB Code 55% faster with GitHub Copilot

Raw

```
2939     flux4(q_im2, q_im1, q_i, q_ip1, ua) = &
2940         ua*( 7.*(q_i + q_im1) - (q_ip1 + q_im2) )/12.0
2941
2942     flux3(q_im2, q_im1, q_i, q_ip1, ua, coef3) = &
2943         flux4(q_im2, q_im1, q_i, q_ip1, ua) + &
2944         coef3*abs(ua)*((q_ip1 - q_im2)-3.*(q_i-q_im1))/12.0
2945
2946     local_advance_density = advance_density
```

$$F(u, \psi)_{i+1/2} = u_{i+1/2} \left[\frac{1}{2} (\psi_{i+1} + \psi_i) - \frac{1}{12} (\delta_x^2 \psi_{i+1} + \delta_x^2 \psi_i) + \text{sign}(u) \frac{\beta}{12} (\delta_x^2 \psi_{i+1} - \delta_x^2 \psi_i) \right]$$

where $\delta_x^2 \psi_i = \psi_{i-1} - 2\psi_i + \psi_{i+1}$ (Hundsdoerfer et al, 1995; Van Leer, 1985)

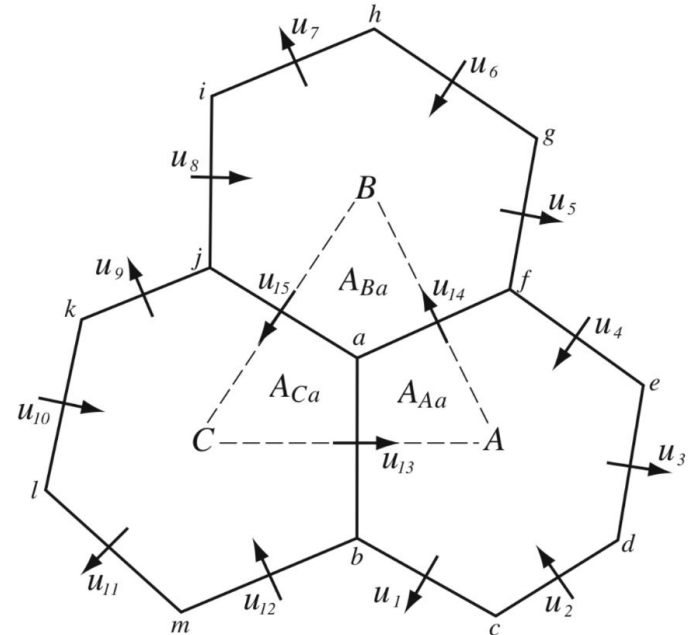
Nonlinear Coriolis Term

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_\zeta \left(\frac{p}{\zeta_z} \right) - \frac{\partial \mathbf{z}_{HP}}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H$$

$$- \mathbf{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K - eW \cos \alpha_r - \frac{uW}{r_e} + \mathbf{F}_{V_H},$$

Vorticity is computed by evaluating the circulation around the triangles.
 Vorticity *lives* on the vertices.

First, the linear piece: $f \mathbf{k} \times \mathbf{V}_H$



Nonlinear Coriolis Term

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_\zeta \left(\frac{p}{\zeta_z} \right) - \frac{\partial z_{HP}}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H$$

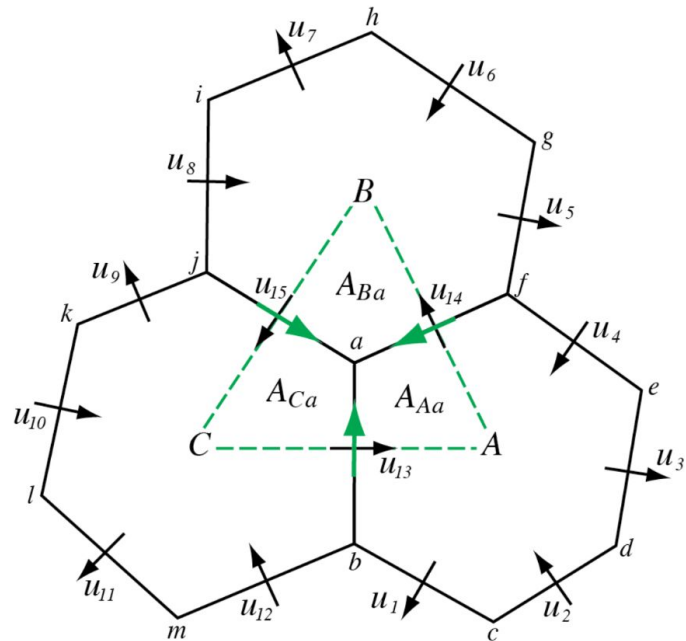
$$- \mathbf{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K - eW \cos \alpha_r - \frac{uW}{r_e} + \mathbf{F}_{V_H},$$

Vorticity is computed by evaluating the circulation around the triangles.

Vorticity *lives* on the vertices.

First, the linear piece: $f \mathbf{k} \times \mathbf{V}_H$

How do we compute the tangential velocity on the cell faces needed in the Coriolis term?



Nonlinear Coriolis Term

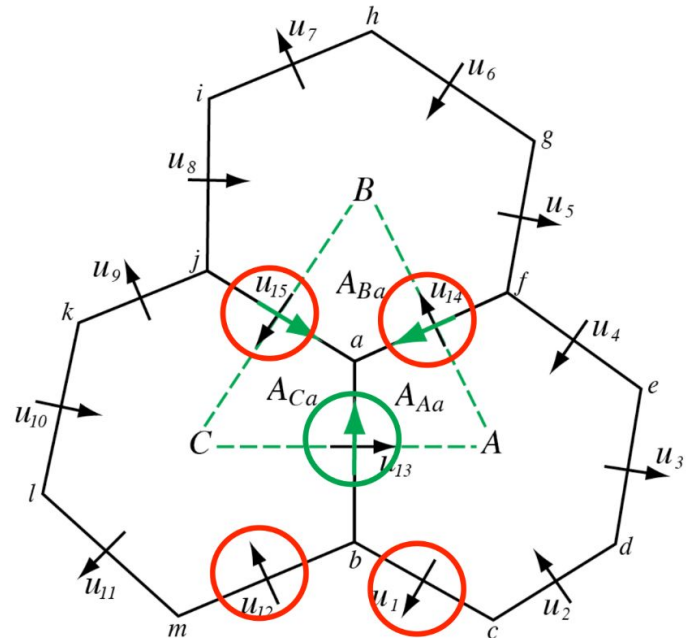
$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_\zeta \left(\frac{p}{\zeta_z} \right) - \frac{\partial \mathbf{z}_{HP}}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H$$

$$- \mathbf{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K - eW \cos \alpha_r - \frac{uW}{r_e} + \mathbf{F}_{V_H},$$

Linear piece: $f \mathbf{k} \times \mathbf{V}_H$

Simplest approach: Construct tangential velocities from weighted sum of the four nearest neighbors.

Result: physically stationary geostrophic modes (geostrophically-balanced flow) will not be stationary in the discrete system; the solver is unusable.



Nonlinear Coriolis Term

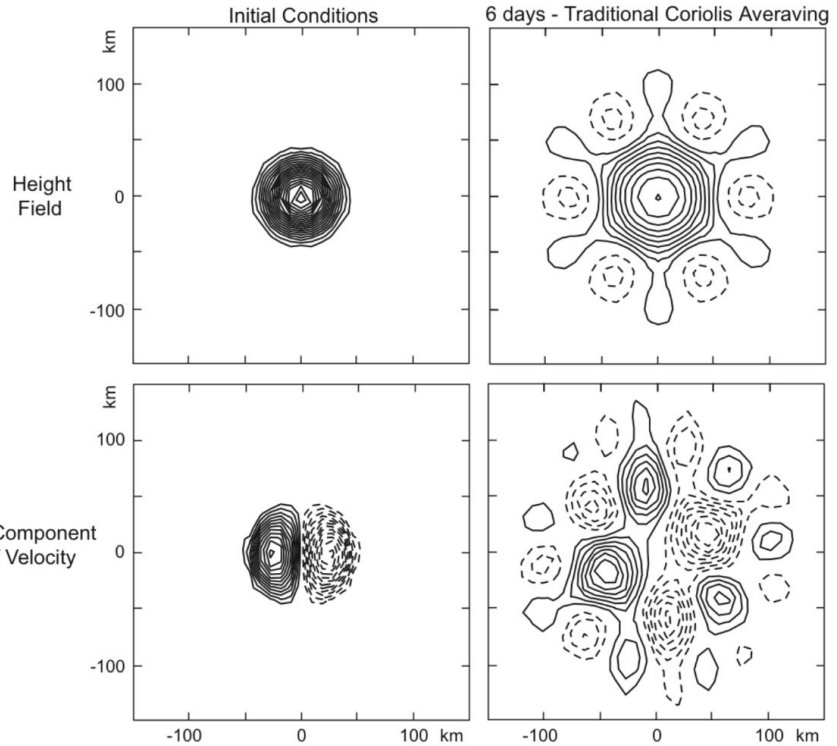
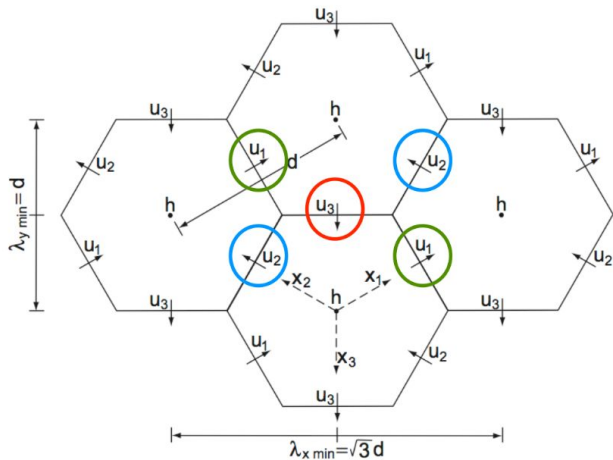
Linear piece: $f \mathbf{k} \times \mathbf{V}_H$

$$\partial_t u_1 + g \delta_{x_1} h + \frac{f}{\sqrt{3}}(u_{31} - u_{21}) = 0$$

$$\partial_t u_2 + g \delta_{x_2} h + \frac{f}{\sqrt{3}}(u_{12} - u_{32}) = 0$$

$$\partial_t u_3 + g \delta_{x_3} h + \frac{f}{\sqrt{3}}(u_{23} - u_{13}) = 0$$

$$\partial_t h + \frac{2}{3} H (\delta_{x_1} u_1 + \delta_{x_2} u_2 + \delta_{x_3} u_3) = 0$$



Nonlinear Coriolis Term

(Thurnburn et al, 2009 JCP)

Linear piece: $f \times V_H$

$$\partial_t u_1 + g \delta_{x_1} h + \frac{f}{\sqrt{3}} (u_{31} - u_{21}) = 0$$

$$\partial_t u_2 + g \delta_{x_2} h + \frac{f}{\sqrt{3}} (u_{12} - u_{32}) = 0$$

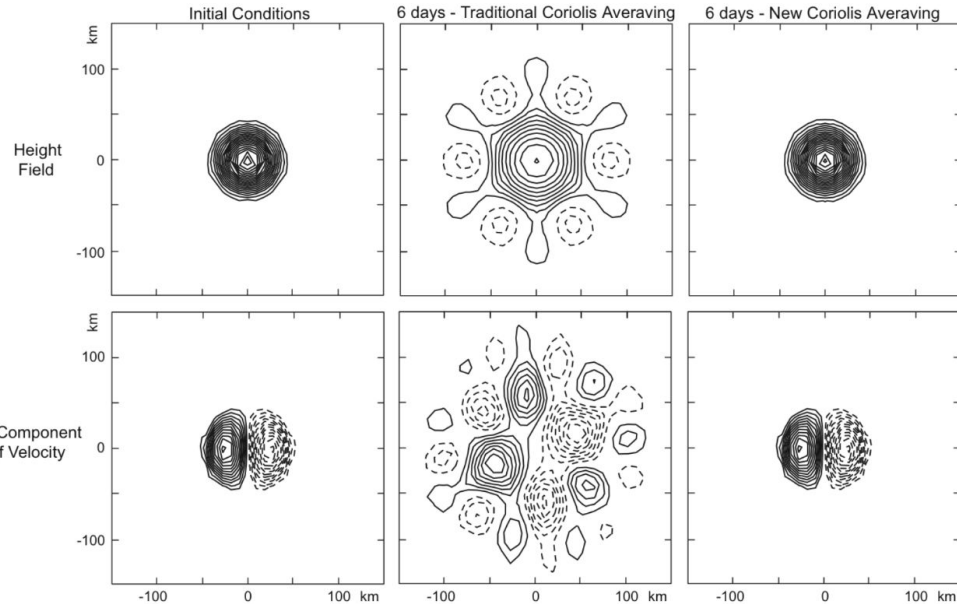
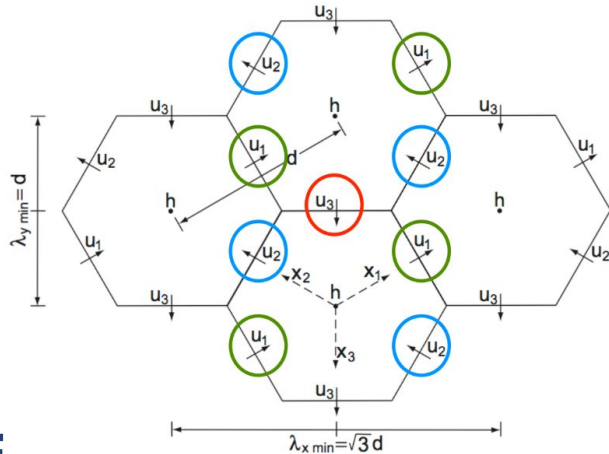
$$\partial_t u_3 + g \delta_{x_3} h + \frac{f}{\sqrt{3}} (u_{23} - u_{13}) = 0$$

$$\partial_t h + \frac{2}{3} H (\delta_{x_1} u_1 + \delta_{x_2} u_2 + \delta_{x_3} u_3) = 0$$

$$u_{21} = \frac{1}{3} \overline{u_2}^{x_3} + \frac{2}{3} \overline{u_2}^{x_1 x_2}, \quad u_{31} = \frac{1}{3} \overline{u_3}^{x_2} + \frac{2}{3} \overline{u_3}^{x_1 x_3}$$

$$u_{12} = \frac{1}{3} \overline{u_1}^{x_3} + \frac{2}{3} \overline{u_1}^{x_1 x_2}, \quad u_{32} = \frac{1}{3} \overline{u_3}^{x_1} + \frac{2}{3} \overline{u_3}^{x_2 x_3}$$

$$u_{13} = \frac{1}{3} \overline{u_1}^{x_2} + \frac{2}{3} \overline{u_1}^{x_1 x_3}, \quad u_{23} = \frac{1}{3} \overline{u_2}^{x_1} + \frac{2}{3} \overline{u_2}^{x_2 x_3}$$



Nonlinear Coriolis Term

In the discrete analogue of vorticity equation ($\xi_{\tau} = -f\delta_a$), the divergence δ_a on the Delaunay triangulation is identical to the divergence δ_A on the Voronoi hexagons used in the height equation ($h_t = -H\delta_A$) integrated over the triangle.

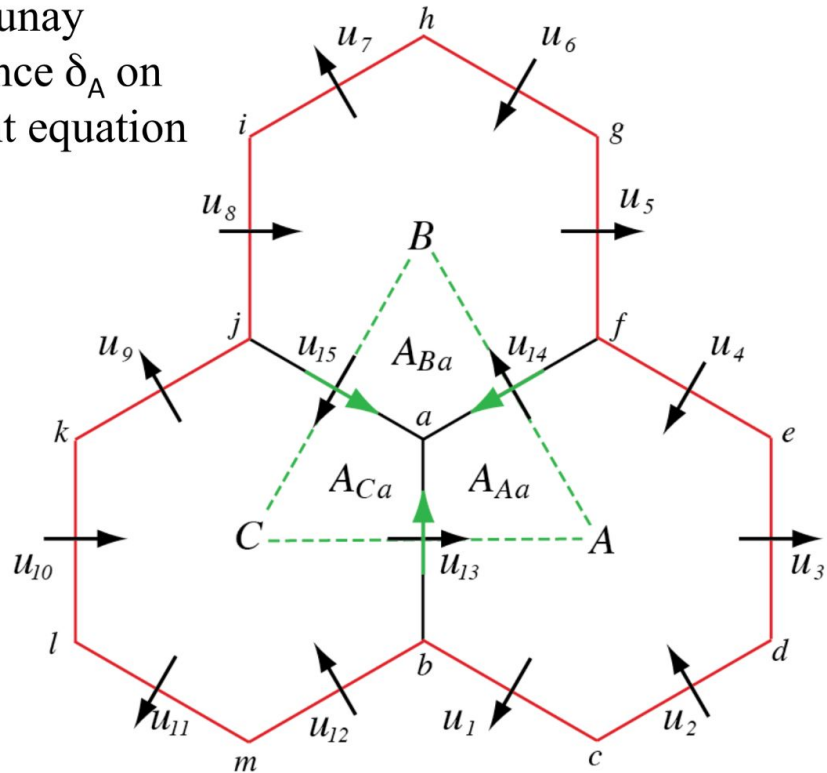
$$A_a \delta_a = \frac{A_A \delta_A + A_B \delta_B + A_C \delta_C}{6}$$

Divergence δ_A in hexagon A:

$$A_A \delta_A = \sum_{i=1}^6 l_i u_i \cdot \mathbf{n}_i$$

Divergence δ_a in triangle ABC:

$$A_a \xi_t = -f A_a \delta_a = f \sum_{j=1}^3 d_j u_j^{\perp} \cdot \mathbf{n}_j$$



Nonlinear Coriolis Term

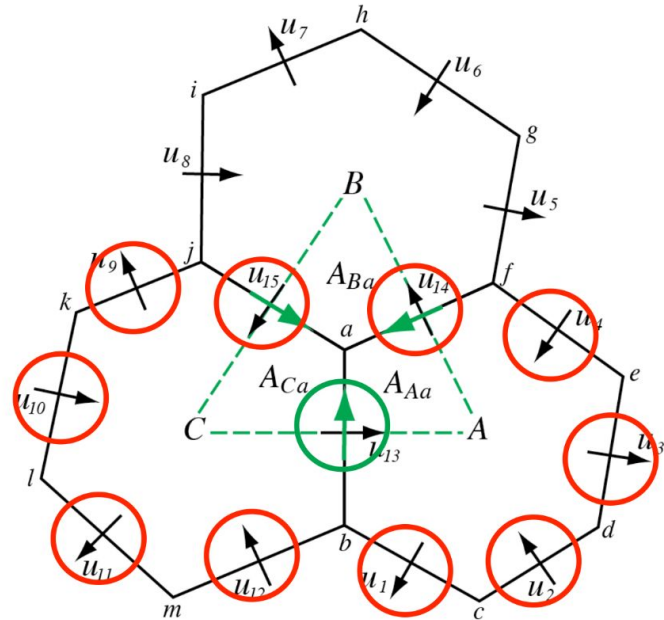
Linear piece: $f \mathbf{k} \times \mathbf{V}_H$

Generalization for the Voronoi mesh:

Construct tangential velocities from weighted sum of normal velocities on edges of adjacent hexagons.

$$d_e u_e^\perp = \sum_j w_e^j l_j u_j$$

Result: geostrophic modes are stationary;
local and global mass and PV conservation is
satisfied on the dual (triangular) mesh (for the
SW equations).



Nonlinear Coriolis Term

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m} \left[\nabla_\zeta \left(\frac{p}{\zeta_z} \right) - \frac{\partial z_{HP}}{\partial \zeta} \right] - \eta \mathbf{k} \times \mathbf{V}_H$$

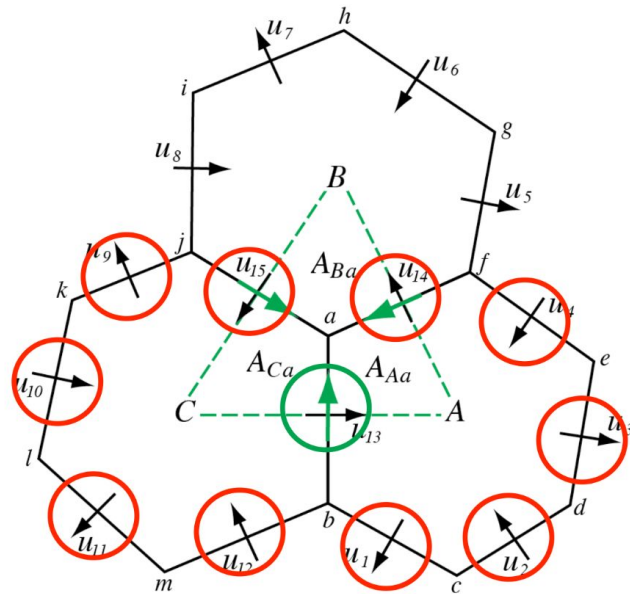
$$- \mathbf{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K - eW \cos \alpha_r - \frac{uW}{r_e} + \mathbf{F}_{V_H},$$

Nonlinear term:

$$\mathbf{v}_{e_i} = \sum_{j=1}^{n_{e_i}} w_{e_{i,j}} \mathbf{u}_{e_{i,j}}$$

$$[\eta \mathbf{k} \times \mathbf{V}_H]_{e_i} = \sum_{j=1}^{n_{e_i}} \frac{1}{2} (\eta_{e_i} + \eta_{e_{i,j}}) w_{e_{i,j}} \rho_{e_{i,j}} \mathbf{u}_{e_{i,j}}$$

The general tangential velocity reconstruction produces a consistent divergence on the primal and dual grids, and allows for PV, enstrophy and energy* conservation in the nonlinear SW solver.



Exemplo código (atmosphere)

MPAS-BR / src / core_atmosphere / dynamics / mpas_atm_time_integration.F

Code Blame 6607 lines (5328 loc) · 293 KB  Code 55% faster with GitHub Copilot

```
4410
4411     ! Next, nonlinear Coriolis term (q) following Ringler et al JCP 2009
4412
4413     q(:) = 0.0
4414     do j = 1, nEdgesOnEdge(iEdge)
4415         eoe = edgesOnEdge(j, iEdge)
4416         do k=1, nVertLevels
4417             workpv = 0.5 * (pv_edge(k, iEdge) + pv_edge(k, eoe))
4418             ! the original definition of pv_edge had a factor of 1/density. We have removed that factor
4419             ! given that it was not integral to any conservation property of the system
4420             q(k) = q(k) + weightsOnEdge(j, iEdge) * u(k, eoe) * workpv
4421         end do
4422     end do
```